COMMUNICATIONS IN COMBINATORICS

Hyatt on the Bund

Shanghai, China

December 12-15, 2025

COMMUNICATIONS IN COMBINATORICS

December 12-15, 2025

Organizing Committee

Hong Liu, Institute for Basic Science

Hehui Wu, Fudan University

Changxiang He, University of Shanghai for Science and Technology

Suyun Jiang, Jianghan University

Xiaowei Yu, Jiangsu Normal University & Institute for Basic Science

Zixiang Xu, Institute for Basic Science

Schedule

- **December 12, Friday**

  Registration day
  - 15:00-18:00 Registration
  - 18:00 -21:00 Dinner

- **December 13, Saturday**

  Opening Ceremony
  - 8:30-9:00

  Session 13A Problem Presentations
  - 9:00-9:10 Problem 13A-1
  - 9:10-9:20 Problem 13A-2
  - 9:20-9:30 Problem 13A-3
  - 9:30-9:40 Problem13A-4
  - 9:40-9:50 Problem 13A-5
  - 9:50-10:00 Problem 13A-6

  Coffee Break & Group Photo
  - 10:00-10:30

  Session 13B Problem Presentations
  - 10:30-10:40 Problem 13B-1
  - 10:40-10:50 Problem 13B-2
  - 10:50-11:00 Problem 13B-3
  - 11:00-11:10 Problem 13B-4
  - 11:10-11:20 Problem 13B-5
  - 11:20-11:30 Problem 13B-6

  Lunch
  - 11:30-13:30

  Session 13C Problem Presentations
  - 14:00-14:10 Problem 13C-1
  - 14:10-14:20 Problem 13C-2
  - 14:20-14:30 Problem 13C-3
  - 14:30-14:40 Problem 13C-4

- 14:40-14:50 Problem 13C-5
- 14:50-15:00 Problem 13C-6

Coffee Break

- 15:00-15:30

Session 13D Discussions

- 15:30-18:00 Free discussion

Banquet

- 18:00-21:00

- ## December 14, Sunday

Session 14A Problem Presentations

- 9:00-9:10 Problem 14A-1
- 9:10-9:20 Problem 14A-2
- 9:20-9:30 Problem 14A-3
- 9:30-9:40 Problem 14A-4
- 9:40-9:50 Problem 14A-5
- 9:50-10:00 Problem 14A-6

Coffee Break

- 10:00-10:30

Session 14B Problem Presentations

- 10:30-10:40 Problem 14B-1
- 10:40-10:50 Problem 14B-2
- 10:50-11:00 Problem 14B-3
- 11:00-11:10 Problem 14B-4
- 11:10-11:20 Problem 14B-5
- 11:20-11:30 Problem 14B-6

Lunch

- 11:30-13:30

Session 14C Discussions

- 14:00-18:00 Free discussion

- ## December 15, Monday

Session 15A Discussions

- 9:00-11:30 Free discussion

Planar graph is odd 5-colorable

Proposed by Ilkyoo Choi, Hankuk University of Foreign Studies

A graph is odd k-colorable if it has a proper k-coloring where every vertex has a color appearing an odd number of times on its neighborhood. Petruševski and Škrekovski conjectured that every planar graph is odd 5-colorable. It is known that every planar graph is odd 8-colorable.

On degree condition for connectivity keeping paths in k-connected triangle-free graphs

Proposed by Shinya Fujita, Yokohama City University

Let m, k be integers with $m \geq 1, k \geq 2$. For a k-connected graph G, a subgraph H of G is k-*removable* if $G - V(H)$ is still a k-connected graph. A graph is *triangle-free* if it contains no triangle as a subgraph.

In [1], I showed that if G is a k-connected triangle-free graph with minimum degree at least $k + (m-1)/2$, then for any vertex $v \in V(G)$, there exists a path P on m vertices starting from v such that $G - V(P)$ is a $(k-1)$-connected graph.

Considering the complete bipartite graph $K_{k+(m-1)/2-1,k+(m-1)/2-1}$ for odd $m \geq 3$, we see that the minimum degree bound is best possible. But we do not know the sharpness for even m in this result.

Can we improve the minimum degree condition for even m? Or, does there exist a construction showing the sharpness of the minimum degree bound "$k + m/2$" for even m? I would like to propose this as an open question.

References

[1] S. Fujita, Connectivity keeping paths containing prescribing vertices in highly connected triangle-free graphs, *J. Combin. Theory Ser. B* **174**:190–206 (2025).

List coloring of 3-chromatic planar graphs

Proposed by Masaki Kashima, Keio University

It is well known that every planar graph is 5-choosable and every planar bipartite graph is 3-choosable. As every planar graph is 4-colorable and every bipartite graph is 2-colorable, it is natural to ask whether every 3-colorable planar graph is 4-choosable.

However, Voigt and Wirth [2] constructed a 3-colorable planar graph which is not 4-choosable. Then, what is a sufficient condition for a 3-chromatic planar graph to be 4-choosable? We pose the following problem.

Problem 1. *Is every planar graph that can be partitioned into an independent set and an induced forest 4-choosable?*

Obviously, if G has a vertex partition into an independent set and an induced forest, then G is 3-colorable. We remark that the following lemma implies that the statement is true if the forest is a matching.

Lemma 2 (Kostochka and Yancy [1])**.** *Let G be a graph and let A be an independent set of G. Let D be a digraph obtained from G by replacing each edge in $G - A$ by a pair of symmetric arcs and orienting each edge between A and $V(G) \setminus A$ arbitrary. Then D is kernel-perfect.*

This problem is also related to the notion of λ-choosability, which was introduced by Zhu [3].

References

[1] A. Kostochka and M. Yancy, Ore's conjecture on color-critical graph is almost true, *J. Combin. Theory Ser. B* 109 (2014), 73–101.

[2] M. Voigt and B. Wirth, On 3-colorable non 4-choosable planar graphs, *J. Graph Theory* 24 (3) (1997), 233–235.

[3] X. Zhu, A refinement of choosability of graphs, *J. Combin. Theory Ser. B* 141 (2020), 143–164.

Chromatic number of square of cubic bipartite planar graph

Proposed by Seog-Jin Kim, Konkuk University

Dvořák et. al [2] and Feder et. al [3] conjectured the following.

Conjecture 1. *Is it true that* $\chi(G^2) \leq 6$ *if* G *is a cubic bipartite planar graph?*

This bound is tight, as exemplified by the hexagonal prism. Feder et al. [3] verified Conjecture 1 in special cases, showing that $\chi(G^2) \leq 6$ if the faces of a bipartite cubic plane graph can be three-colored red, blue, and green such that red faces have even size and blue and green faces have sizes divisible by four.

As a natural direction, we can ask the following question.

Problem 2. *Is it true that* $\chi_\ell(G^2) \leq 6$ *if* G *is a cubic bipartite planar graph?*

References

[1] D. W. Cranston, Coloring, List Coloring, and Painting Squares of Graphs (and Other Related Problems), Electron. J. Combin. 30(2) (2023), #DS25.

[2] Z. Dvořák, R. Škrekovski, and M. Tancer, List-coloring squares of sparse subcubic graphs, SIAM J. Discrete Math. 22 (2008), no. 1, 139–159

[3] T. Feder, P. Hell, and C. Subi, Distance-two colourings of Barnette graphs, European J. Combin. 91 (2021), Paper No. 103210.

Saturation numbers of cycles

Proposed by Younjin Kim, POSTECH

Given a graph F, a graph G is said to be F-*saturated* if G is F-free and, for any two nonadjacent vertices x and y of G, the graph $G + xy$ contains a copy of F. The *saturation number* of F, denoted by $\mathrm{sat}(n, F)$, is the minimum number of edges in an F-saturated graph on n vertices. Tuza [4] conjectured that for every graph F, there exists a constant c_F such that $\mathrm{sat}(n, F) = c_F n + O(1)$.

Let C_k denote the cycle graph on k vertices. Füredi and Kim [2] established the bounds

$$\frac{k+3}{k+2}n - 1 < \mathrm{sat}(n, C_k) < \frac{k-3}{k-4}n + \binom{k-4}{2}$$

for $k \geq 7$ and $n \geq 2k - 5$, and proposed the following conjecture.

Conjecture 1 (Füredi–Kim, 2013). *There exists a constant* k_0 *such that*

$$\mathrm{sat}(n, C_k) = \frac{k-3}{k-4}n + O(k^2)$$

holds for all integers $k \geq k_0$.

Recently, Mohammadian, Poursoltani, and Tayfeh-Rezaie [3] confirmed this for even cycles of length at least 28.

Theorem 2 ([3]). *For each fixed even integer* $k \geq 28$, $\mathrm{sat}(n, C_k) = \frac{k-3}{k-4}n + O(1)$.

This verifies Tuza's conjecture for even cycles of length at least 28. However, the following cases remain open.

Problem 3. *Prove that* $\mathrm{sat}(n, C_k) = \frac{k-3}{k-4}n + O(1)$ *for even integers* $8 \leq k \leq 26$.

Problem 4. *Prove that* $\mathrm{sat}(n, C_k) = \frac{k-3}{k-4}n + O(1)$ *for odd integers* $k \geq 7$.

Note that C_6 is exceptional: Lan et al. [1] proved $\mathrm{sat}(n, C_6) = \frac{4}{3}n + O(1) \neq \frac{3}{2}n + O(1)$.

References

[1] Y. Lan, Y. Shi, Y. Wang, J. Zhang. The saturation number of C_6. *Discrete Math.* 348 (2025), 114504.

[2] Z. Füredi, Y. Kim. Cycle-saturated graphs with minimum number of edges. *J. Graph Theory* 73 (2013), 203–215.

[3] A. Mohammadian, M. Poursoltani, B. Tayfeh-Rezaie. On saturation numbers of complete multipartite graphs and even cycles. *arXiv:2506.09767*, 2025.

[4] Zs. Tuza. Asymptotic growth of sparse saturated structures is locally determined. *Discrete Math.* 108 (1992), 397–402.

Lower bounds for semiproper colourings

Proposed by Joonkyung Lee, Yonsei University

Amongst d-regular graphs, which one has fewest independent sets? One may easily guess that the answer should be the complete graph K_{d+1}. Indeed, this is a theorem of Cutler and Radcliffe [1], stating that

$$i(G)^{1/v(G)} \geq i(K_{d+1})^{1/(d+1)}$$

holds for any d-regular graph G, where $i(G)$ denotes the number of independent sets in a graph G. Sah, Sawhney, Stoner and Zhao [2] generalised this to a possibly irregular graphs G.

An analogous minimisation question can also be asked for the number of q-colourings, for which the answer is again the complete graph K_{d+1}. Namely, Csikvári's theorem, recorded in Zhao's survey [3], states that

$$c_q(G)^{1/v(G)} \geq c_q(K_{d+1})^{1/(d+1)}$$

holds for any d-regular graph G, where $c_q(G)$ denotes the number of proper q-colourings of G. This also generalises to irregular graphs too, by a minor modification of the proof.

Having seen the similarity of these results, it is natural to look for a common generalisation, as follows. Let $K_q^{\ell\circ}$ be the graph obtained by adding ℓ self-loops to K_q without making multi-loops. Then $i(G) = \hom(G, K_2^{\ell\circ})$ with $\ell = 1$ and $c_q(G) = \hom(G, K_q^{\ell\circ})$ with $\ell = 0$. This is the so-called *semiproper colourings*, where the looped vertices correspond to 'free' colours that can be used without restrictions. For each $0 \leq \ell \leq q$, we conjecture that

$$\hom(G, K_q^{\ell\circ})^{1/v(G)} \geq \hom(K_{d+1}, K_q^{\ell\circ})^{1/(d+1)}$$

holds for any d-regular graph G. The first interesting case, $q = 3$ and $\ell = 1$ even remains open.

References

[1] Jonathan Cutler and Andrew J. Radcliffe, *The maximum number of complete subgraphs of fixed size in a graph with given maximum degree.* J. Graph Theory 84.2 (2017), pp. 134–145.

[2] Ashwin Sah, Mehtaab Sawhney, David Stoner, and Yufei Zhao, *The number of independent sets in an irregular graph.* J. Combin. Theory Ser. B 138 (2019), pp. 172–195.

[3] Yufei Zhao, *Extremal regular graphs: independent sets and graph homomorphisms.* Amer. Math. Monthly 124.9 (2017), pp. 827–843.

List packing number on d-trees

Proposed by Shun-ichi Maezawa, Nihon University

For a graph G, a *list assignment* L of G is a mapping such that each $v \in V(G)$ is assigned a list of colors $L(v)$. For a positive integer k, a *k-list assignment* is a list assignment for which each list has size k. An *L-coloring* c is a proper coloring such that $c(v) \in L(v)$ for $v \in V(G)$. For a list-assignment L of a graph G, an *L-packing of G of size* k is a collection of k mutually disjoint L-colorings $c_1, c_2, \ldots, c_k$ of G, that is, $c_i(v) \neq c_j(v)$ for any distinct $i, j \in \{1, 2, \ldots, k\}$ and any $v \in V(G)$. The list packing number $\chi_\ell^*(G)$ of a graph G is the maximum integer k such that G admits a L-packing of size k for any list assignment L of G. Cambie et al. [1] proved that $d + 2 \leq \chi_\ell^*(G) \leq 2d$ for every d-degenerate graph G, where a graph G is d-degenerate if every induced subgraph of G has a vertex of degree at most d.

A graph G is a d-tree if there is a sequence $K_{d+1} = G_0, G_1, \ldots, G_m = G$ of induced subgraphs of G such that for every integer $1 \leq i \leq m$, there is a vertex $v_{i+d+1} \in V(G_i) \setminus V(G_{i-1})$ satisfying that $V(G_i) = V(G_{i-1}) \cup \{v_{i+d+1}\}$ and $N_G(v_{i+d+1}) \cap V(G_{i-1})$ is a clique of order d of G. Note that a d-tree is a d-degenerate graph. Vey recently, Kashima, Zhu, and I have proved that $d + 2 \leq \chi_\ell^*(G) \leq 2d - 1$ for every d-tree G ($d \geq 2$). Motivated by the gap between these bounds ($d + 2$ and $2d - 1$), it is natural to consider the following problem: What is the exact coefficient of the list packing number for d-trees in terms of d? Moreover, focusing on improving the upper bound, we may also consider the following problem: How many distinct L-colorings can be found in d-trees with $(2d - 2)$-list assignment?

References

[1] S. Cambie, W. Cames van Batenburg, E. Davis, R. J. Kang, Packing list-colorings, Random Struct. Alg. **64** (2024), 62–93.

On the minimum spectral radius in an n-vertex K_{r+1}-saturated graph

Proposed by Suil O, The State University of New York, Korea

Given two graphs G and H, a graph G is *H-saturated* if G does not contain H as a subgraph, but for any $e \in E(\overline{G})$, $G + e$ contains H as a subgraph; the *spectral saturation number* of H, written $sat_\rho(n, H)$, is the minimum value of $\rho(G)$ in an n-vertex H-saturated graph G.

In 2020, Kim, Kim, Kostochka, and O [2] conjectured a lower bound for the spectral radius in an n-vertex K_{r+1}-saturated graph.

Conjecture 1. ([2]) *For $n \geq r + 1$ and $r \geq 3$, we have $sat_\rho(n, K_{r+1}) = \rho(S_{n,r})$, where $S_{n,r} = K_{r-1} \vee \overline{K_{n-r+1}}$.*

Kim, Kim, Kostochka, and O [2] proved a lower bound for $sat_\rho(n, K_{r+1})$ and determined $sat_\rho(n, K_3)$. In 2023, Kim, Kostochka, O, Shi, and Wang determined $sat(n, K_4)$. Very recently, Ai, Liu, O, and Zhang [1] determined $sat_\rho(n, K_5)$ and $sat_\rho(n, tP_3)$ for a positive integer. Wang and Hou also proved $sat_\rho(n, K_5)$ as well as $sat_\rho(n, K_6)$.

Possible Tool

If we can prove the following conjecture, then Conjecture 1 can be settled down.

Conjecture 2. (O) *If G is an n-vertex K_{r+1}-saturated graph, then for each vertex $v \in V(G)$, we have*

$$\sum_{w \in N(v)} d(w) \geq (r-2)d(v) + (r-1)(n-r+1).$$

Conjecture 2 is true for $r = 3, 4, 5$ and was used to prove Conjecture 1 for $r = 3, 4, 5$.

References

[1] J. Ai, P. Liu, S. O, and J. Zhang, The Minimum Spectral Radius of tP_3- or K_5-Saturated Graphs via the Number of 2-Walks *Electronic J. Combin.* **32** (2025), P1.30.

[2] J. Kim, S. Kim, A.V. Kostochka, and S. O, The minimum spectral radius of K_{r+1}-saturated graphs , *Discrete Math.* **343**(2020), 112068.

[3] J. Kim, A.V. Kostochka, S. O, Y. Shi, and Z. Wang, A sharp lower bound for the spectral radius in K_4-saturated graphs, *Discrete Math.* **346**(2023), 113231.

[4] D. Wang and Y. Hou, The minimum spectral radius for K_{r+1}-saturated graphs with $r = 4, 5$, *Discrete Math.* **347**(2024), 114110.

Extremal graphs avoiding cycles of length ℓ modulo k

Proposed by Boram Park, Seoul National University

For two integers k and ℓ, an (ℓ mod k)-cycle or (ℓ mod k)-path means a cycle or path of length m such that $m \equiv \ell \pmod{k}$. Burr and Erdős [7] conjectured that if an n-vertex graph does not contain an (ℓ mod k)-cycle, where $k\mathbb{Z} + \ell$ contains an even number, then it has at most a linear number of edges in terms of n. This conjecture was later confirmed by Bollobás [3], whose result naturally led to the following question: What is the smallest constant $c_{\ell,k}$, where $k\mathbb{Z} + \ell$ contains an even number, such that every n-vertex graph with $c_{\ell,k}n$ edges contains an (ℓ mod k)-cycle? Sudakov and Verstraëte [11] showed that for $3 \leq \ell < k$, the value of $c_{\ell,k}$ is proportional to the maximum average degree of a k-vertex graph without cycles of length ℓ. This determines $c_{\ell,k}$ up to an absolute constant. The exact value of $c_{\ell,k}$ is known only for a few specific values of ℓ and k.

- It is well known that $c_{0,2} = \frac{3}{2}$.
- Chen and Saito [4] proved that $c_{0,3} = 2$ and the extremal graphs are $K_{2,n-2}$.
- Bai, Li, Pan and Zhang [2] proved that $c_{1,3} = \frac{5}{3}$, and one vertex identifications of Petersen graphs are extremal graphs.
- Dean, Kaneko, Ota and Toft [6], as well as Saito [10], showed that $c_{2,3} = 3$, with extremal graphs being $K_{3,n-3}$.
- Gao, Li, Ma and Xie [8] proved that an n-vertex graph with at least $\frac{5(n-1)}{2}$ edges contains two consecutive even cycles unless $4 \mid (n-1)$ and every block is isomorphic to K_5. This result not only shows that $c_{2,4} = \frac{5}{2}$.
- In [9], Győri, Li, Salia, Tompkins, Varga and Zhu determined that $c_{0,4} = \frac{19}{12}$.
- Most Recently in [1], Bai, Grzesik, Li, Prorok showed that $c_{0,k} = k - 1$ for all odd integer k.

Problem 1. *What is $c_{\ell,k}$ in general when $k\mathbb{Z} + \ell$ contains an even integer?*

We can ask the same question when G has a certain extra condition, such as 2-connectivity. See [5].

Problem 2. *On the class of 2-connected graphs, what is $c_{\ell,k}$ in general when $k\mathbb{Z} + \ell$ contains an even integer?*

References

[1] Y. Bai, A. Grzesik, B. Li, M. Prorok, Cycle lengths in graphs of given minimum degree, *arXiv*: 2511.03085v1.

[2] Y. Bai, B. Li, Y. Pan and S. Zhang, On graphs without cycles of length 1 modulo 3, *arXiv*: 2503:03504.

[3] B. Bollobás. Cycles modulo k, *Bulletin of the London Mathematical Society*, 9(1):97–98, 1977.

[4] G. T. Chen and A. Saito, Graphs with a cycle of length divisible by three, *Journal of Combinatorial Theory, Series B*, 60(2):277–292, 1994.

[5] H. Chu, B. Park, and H. Ryu, On 2-connected graphs avoiding cycles of length 0 modulo 4 *arXiv*: 2507:12798

[6] N. Dean, A. Kaneko, K. Ota, and B. Toft, Cycles modulo 3, *Dimacs Technical Report*, 91(32), 1991.

[7] P. Erdős, Some recent problems and results in graph theory, combinatorics, and number theory, In *Proc. Seventh SE Conf. Combinatorics, Graph Theory and Computing, Utilitas Math*, 3–14, 1976.

[8] J. Gao, B. Li, J. Ma, and T. Xie, On two cycles of consecutive even lengths, *Journal of Graph Theory*, 106(2):225–238, 2024.

[9] E. Győri, B. Li, N. Salia, C. Tompkins, K. Varga, and M. Zhu, On graphs without cycles of length 0 modulo 4, *arXiv*:2312.09999.

[10] A. Saito, Cycles of length 2 modulo 3 in graphs, *Discrete Mathematics*, 101(1–3):285–289, 1992.

[11] B. Sudakov and J. Verstraëte, The extremal function for cycles of length ℓ mod k, *The Electronic Journal of Combinatorics*, 24(1), 2017.

The degree of asymmetry of graphs and tournaments

Proposed by Shohei Satake, Kumamoto University

In [3], Erdős and Rényi investigated the *degree of asymmetry* of a graph G, namely,

$$A(G) := \min\left\{|S| : S \subset \binom{V(G)}{2} \text{ s.t. } |\mathrm{Aut}(G \triangle S)| > 1\right\}$$

where $G \triangle S$ is a graph on $V(G)$ with edge set $E(G) \triangle S$, and Aut denotes the full automorphism group. They proved that random graph $\mathcal{G}(n) := \mathcal{G}(n, 1/2)$ satisfies

$$A(\mathcal{G}(n)) \geq \frac{n}{2} - O(\sqrt{n \log n})$$

w.h.p. For an integer $n \geq 1$, let $A(n) := \max\{A(G) : G \text{ is an } n\text{-vertex graph}\}$. They also showed that

$$A(n) \leq \frac{n-1}{2}$$

implying that almost all graphs are "extremely" asymmetric. These results have affected many works in extremal, probabilistic and algebraic combinatorics, as well as in theoretical computer science, see e.g. [1, 2, 4, 5, 7] and references therein.

In [6], we extend the notion of degree of asymmetry to *tournaments* (i.e. oriented complete graphs). For a tournament T, let

$$B(T) := \min\left\{|S| : S \subset V(T) \times V(T) \text{ s.t. } S \text{ is inverse-closed and } |\mathrm{Aut}(T \triangle S)| > 1\right\}.$$

where $S \subset V(T) \times V(T)$ is said to be *inverse-closed* if $(u, v) \in S$ implies $(v, u) \in S$, and $T \triangle S$ is defined in a manner similar to the graph case. Let $B(n) := \max\{B(T) : T \text{ is an } n\text{-vertex tournament}\}$. We proved that random tournament $\mathcal{T}(n)$ (sampled from the set of all n-vertex tournaments uniformly at random) satisfies

$$B(\mathcal{T}(n)) \geq \frac{3n}{4} - O(\sqrt{n \log n})$$

w.h.p., while we have

$$B(n) \leq \frac{3n-2}{4} + o(1).$$

We have the following open problems.

Problem 1. *Is it true that*

$$A(\mathcal{G}(n)) = \frac{n}{2} - O(1), \quad B(\mathcal{T}(n)) = \frac{3n}{4} - O(1)$$

hold w.h.p.?

Problem 2. *Provide* explicit *constructions for an n-vertex graph G and an n-vertex tournament T such that*

$$A(G) = \frac{n}{2} - O(1), \quad B(T) = \frac{3n}{4} - O(1).$$

References

[1] L. Babai, P. Erdős, S. M. Selkow, Random graph isomorphism, *SIAM J. Comput.* **9** (1980), 628–635.

[2] B. Bollobás, Random Graphs, Academic, London, 1985.

[3] P. Erdős, A. Rényi, Asymmetric graphs, *Acta Math. Acad. Sci. Hung.* **14** (1963), 295–315.

[4] O. Goldreich, A. Wigderson, Robustly self-ordered graphs: constructions and applications to property testing, In: *Proceedings of the 36th Computational Complexity Conference (CCC 2021)*, pp. 1-74.

[5] J. H. Kim, B. Sudakov, V. H. Vu, On the asymmetry of random regular graphs and random graphs, *Random Struct. Algorithms.* **21** (2002), 216–224.

[6] S. Satake, The asymmetry number of finite tournaments, and some related results, *Graphs and Combin.* **33** (2017), 1433–1442.

[7] B. Sudakov, Robustness of graph properties. In: *Surveys in Combinatorics 2017*, Cambridge University Press, pp. 372–408, 2017.

Decomposing hypergraphs into Hamiltonian cycles

Proposed by Bjarne Schülke, IBS ECOPRO

For $1 \le \ell \le k-1$, the *Dirac constant* $\hbar_\ell^{(k)}$ is defined as the infimum $\alpha \in [0,1]$ such that for every $\varepsilon > 0$ and sufficiently large n, every n-vertex k-graph H with $\delta_\ell(H) \ge (\alpha+\varepsilon)\binom{n-\ell}{k-\ell}$ contains a Hamiltonian cycle. Let $\mathrm{reg}_k(H)$ be the largest integer r divisible by k such that H contains a spanning subgraph in which every vertex belongs to exactly r edges. Further, let $\hbar_\ell^{(k)\mathrm{dec}}$ be the infimum $\alpha \in [0,1]$ such that for every $\varepsilon > 0$ and sufficiently large n, every n-vertex k-graph H with $\delta_\ell(H) \ge (\alpha+\varepsilon)\binom{n-\ell}{k-\ell}$ contains $(1-\varepsilon)\mathrm{reg}_k/k$ edge-disjoint Hamiltonian cycles. Lastly, define $\hbar_\ell^{(k)\mathrm{ex}}$ as the infimum $\alpha \in [0,1]$ such that for every $\varepsilon > 0$ and sufficiently large n, every n-vertex k-graph H with $\delta_\ell(H) \ge (\alpha+\varepsilon)\binom{n-\ell}{k-\ell}$ can be decomposed into Hamiltonian cycles. Joos, Kühn, and Schülke showed that for all $k \ge 2$, $\hbar_{k-1}^{(k)\mathrm{dec}} = \hbar_{k-1}^{(k)}$. Piga and Sanhueza-Matamala showed that $\hbar_2^{(3)} \ne \hbar_2^{(3)\mathrm{ex}}$.

Determine all $1 \le \ell \le k-1$ such that

$$\hbar_\ell^{(k)\mathrm{dec}} = \hbar_\ell^{(k)} \ne \hbar_\ell^{(k)\mathrm{ex}}.$$

Ore-type condition for homeomorphically irreducible spanning trees

Proposed by Shoichi Tsuchiya, Senshu University

Let G be a connected graph of order n. A spanning tree T of G is called a *homeomorphically irreducible spanning tree* (a *HIST*) if T has no vertices of degree 2. In [1], it was proved that if $\delta(G) \geq 4\sqrt{2n}$, then G has a HIST (and this minimum degree was refined in [2]). For the degree-sum condition of nonadjacent vertices ($\sigma_2(G)$), the following theorems were proved.

Theorem 1 (Ito and Tsuchiya [4])**.** *Let G be a connected graph of order $n \geq 8$. If $\sigma_2(G) \geq n-1$, then G has a HIST.*

Theorem 2 (Furuya, Saito and Tsuchiya [2])**.** *Let G be a connected graph of order $n \geq 10$, and suppose that $\sigma_2(G) \geq n-2$. Then G has a HIST if and only if G is not isomorphic to D_n, where D_n is the graph obtained from K_{n-2} and K_2 by adding one edge.*

Theorem 3 (Furuya and Tsuchiya [3])**.** *Let G be a connected graph of order $n \geq 1091$, and suppose that $\sigma_2(G) \geq \frac{n+2}{2}$. Then G has a HIST if and only if G has no blocking set, where a blocking set is a cut set consisting of vertices of degree 2.*

Since there exists D_n, the bound of Theorem 2 is sharp. However, if we add some properties (as Theorems 2 and 3), we may refine this bounds.

Problem 4. *Are there a property P and a number $c > 0$ such that if a connected graph G of order n (n is sufficiently large) satisfies P and $\sigma_2(G) \geq c\sqrt{n}$, then G has a HIST?*

References

[1] M.O. Albertson, D.M. Berman, J.P. Hutchinson and C. Thomassen, Graphs with homeomorphically irreducible spanning trees, J. Graph Theory **14** (1990), 247–258.

[2] M. Furuya, A. Saito and S. Tsuchiya, Refinements of degree conditions for the existence of a spanning tree without small degree stems, Discrete Math. **348** (2025), 114307.

[3] M. Furuya and S. Tsuchiya, A new strategy for finding spanning trees without small degree stems, Electronic Journal of Combinatorics **32** (2025), 3-2-3.

[4] T. Ito and S. Tsuchiya, Degree sum conditions for the existence of homeomorphically irreducible spanning trees, J. Graph Theory **99** (2022), 162–170.

Strong blocking sets

Proposed by Chong Shangguan, Shandong University (Qingdao)

A set of vectors $U \subseteq \mathbb{F}_q^k$ forms a *strong s-blocking set* if the following two conditions hold: (1) these vectors are pairwise linearly independent (i.e., they are projective points in $PG(k-1, q)$); (2) for every subspace $V \subseteq \mathbb{F}_q^s$ of codimension s, $U \cap V$ spans V.

Strong blocking set is an important concept in finite geometry and it is equivalent to *minimal codes* in coding theory, which also has many applications.

Let $f(k, q, s)$ denote the minimum size of a strong s-blocking set in $\mathbb{F}_q^k$. It is known that

$$\frac{(q^{s+1}-1)(k-s)}{q-1} \leq f(k,q,s) \leq \frac{(q^{s+1}-1)(s+1)(k-s-1)}{q-1}.$$

The lower bound is proved by the polynomial method and the upper bound is attained by a randomized construction. For $s = 1$, a better lower bound $f(k, q, 1) \geq c_q(q+1)(k-1)$ with $c_q > 1$ is known. For $s \geq 2$, there is a gap of factor $s + 1$ between the lower and upper bounds.

Question 1. *Close or narrow the gap.*

The above upper bound is probabilistic. Explicit constructions are also very interesting. Using expander graphs and algebraic geometry codes, Alon, Bishnoi, Das, and Neri provided explicit constructions for strong blocking sets. Generalizing their constructions, Bishnoi and Tomon provided explicit constructions for strong s-blocking sets. These constructions are tight in the order of q and k, but with hugh constant as a function of s, i.e., $2^{O(s^2 \log s)}kq^s$.

Question 2. *Improve the constant in the explicit construction.*

Alon et al made use of the *integrity* $\ell(G)$ of a graph G: this is the least integer such that for every $S \subseteq V(G)$, one can find in $G - S$ a connected component of size $\ell(G) - |S|$. Alon et al used the fact that any (n, d, λ)-graph G has large integrity

$$\ell(G) \geq \frac{d-\lambda}{d+\lambda} \cdot v(G).$$

Question 3. *What is the integrity of a 3-graph? Define and use it to improve Bishnoi and Tomon's construction.*

References

[1] Alon, Bishnoi, Das, and Neri, Strong blocking sets and minimal codes from expander graphs, Trans. AMS, 2024.

[2] Bishnoi and Tomon, Explicit constructions of optimal blocking sets and minimal codes, arXiv:2411.10179

Graphs without cycles of length 0 modulo an even integer

Proposed by Yandong Bai, Northwestern Polytechnical University

An (ℓ mod k)-*cycle* refers to a cycle whose length is congruent to ℓ modulo k. Denote by $\mathcal{C}_{\ell \bmod k}$ the set of all (ℓ mod k)-cycles. Burr and Erdős [6] conjectured in 1976 that $ex(n, \mathcal{C}_{\ell \bmod k}) = \Theta(n)$ for $k > \ell \geqslant 0$ when $k\mathbb{Z} + \ell$ contains an even integer. Bollobás [3] confirmed the above conjecture and demonstrated that $ex(n, \mathcal{C}_{\ell \bmod k}) \leqslant ((k+1)^k - 1)n/4k$. Erdős then proposed the problem of determining the smallest constant $c_{\ell,k}$ such that every n-vertex graph with at least $c_{\ell,k} \cdot n$ edges contains an (ℓ mod k)-cycle.

It is not difficult to see that if G contains no (0 mod 2)-cycles then $e(G) \leqslant 3(n-1)/2$ and, for $2|(n-1)$, the equality holds if and only if each block of G is a triangle. This implies that $c_{0,2} = 3/2$. The first non-trivial result on $c_{\ell,k}$ was obtained by Chen and Saito [4], who showed that $c_{0,3} = 2$. Recently, Győri et al. [8] showed that $c_{0,4} = 19/12$. Bai et al. [1] showed that $c_{0,k} = k-1$ for odd k. Also, Dean et al. [5] and, independently, Saito [9] showed that $c_{2,3} = 3$. Gao et al. [7] showed that $c_{2,4} = 5/2$. Bai et al. [2] showed that $c_{1,3} = 5/3$.

Bai et al. [1] conjecture that $c_{0,k} = (k-1)/2$ for even $k \geqslant 6$.

Problem 1. *Whether* $c_{0,k} = (k-1)/2$ *for even* $k \geqslant 6$.

References

[1] Y. Bai, A. Grzesik, B. Li, M. Prorok, Cycle lengths in graphs of given minimum degree arXiv:2511.03085.

[2] Y. Bai, B. Li, Y. Pan, S. Zhang, On graphs without cycles of length 1 modulo 3, arXiv:2503.03504.

[3] B. Bollobás, Cycles modulo k, Bull. London Math. Soc. 9 (1) (1977) 97–98.

[4] G. Chen, A. Saito, Graphs with a cycle of length divisible by three, J. Combin. Theory, Ser. B 60 (2) (1994) 277–292.

[5] N. Dean, A. Kaneko, K. Ota, B. Toft, Cycles modulo 3, Dimacs Technical Report 91 (32) (1991).

[6] P. Erdős, Some recent problems and results in graph theory, combinatorics and number theory, Proceedings of the Seventh Southeastern Conference on Combinatorics, Graph Theory, and Computing (Louisiana State Univ., Baton Rouge, La., 1976), Congress. Numer. XVII (1976) 3–14.

[7] J. Gao, B. Li, J. Ma, T. Xie, On two cycles of consecutive even lengths, J. Graph Theory 106 (2) (2024) 225–238.

[8] E. Győri, B. Li, N. Salia, C. Tompkins, K. Varga, M. Zhu, On graphs without cycles of length 0 modulo 4, J. Combin. Theory, Ser. B 176 (2026) 7–29.

[9] A. Saito, Cycles of length 2 modulo 3 in graphs, Discrete Math. 101 (1992) 285–289.

Total weight choosability of graphs

Proposed by Kecai Deng, Huaqiao University

For a graph $G = (V, E)$, a mapping $w : V \cup E \to \mathbb{R}$ is called a *proper total weighting*, if for each $uv \in E$, $w(u) + \sum_{e \ni u} w(e) \neq w(v) + \sum_{e \ni v} w(e)$.

We say G is (k, k')-*choosable* if: for any list assignment L that assigns to each vertex v a set $L(v)$ of k real numbers and to each edge e a set $L(e)$ of k' real numbers, there exists a proper total weighting w such that $w(z) \in L(z)$ for each $z \in V \cup E$.

Conjecture 1. (Wong and Zhu [4], 2011) *Every nice graph is* $(1, 3)$*-choosable.*

Known results on Conjecture 1:

1. Every nice graph is $(1, 5)$-choosable. (Zhu [6], 2022)
2. Every nice graph admits a proper $\{1, 2, 3\}$-edge weighting. (Keusch [6], 2024)

Conjecture 2. (Wong and Zhu [4] 2011, Przybyło and Woźniak [3] 2011) *Every graph is* $(2, 2)$-*choosable.*

Known results on Conjecture 2:

1. Every graph is $(2, 3)$-choosable. (Wong and Zhu [5], 2016)
2. Every graph is uniform-span $(2, 2)$-choosable. (Deng and Qiu [1], 2025+)

(Here, we say a graph G is *uniform-span* $(2, 2)$*-choosable* if: for any list assignment L that assigns to each $z \in V \cup E$ a set $L(z) = \{a_z, b_z\}$ $(a_z \neq b_z)$ and satisfies $|a_x - b_x| = |a_y - b_y|$ for arbitrary $x, y \in V \cup E$, there exists a proper total weighting w such that $w(z) \in L(z)$ for each $z \in V \cup E$.)

References

[1] K. Deng, H. Qiu, Every graph is uniform-span (2, 2)-choosable: Beyond the 1-2 conjecture, arXiv:2506.14253v8 [math.CO]

[2] R. Keusch, A solution to the 1-2-3 conjecture, J. Comb. Theory, Ser. B 166 (2024) 183-202.

[3] J. Przybyło, M. Woźniak, Total weight choosability of graphs, Electron. J. Comb. 18 (1) (2011), Paper 112, 11pp.

[4] T.-L. Wong, X. Zhu, Total weight choosability of graphs, J. Graph Theory 66 (2011) 198-212.

[5] T.-L. Wong, X. Zhu, Every graph is (2, 3)-choosable, Combinatorica 36 (2014) 121-127.

[6] X. Zhu, Every nice graph is (1, 5)-choosable, J. Comb. Theory, Ser. B 157 (2022) 524-551.

Properly colored Hamiltonian cycles in edge-colored graph

Proposed by Laihao Ding, Central China Normal University

Given an edge-colored graph G, we say that G is k-*bounded* if for any $v \in V(G)$ each color appears on at most k incident edges of v. Bollobás and Erdős [1] conjectured the following.

Conjecture 1 ([1]). *Every n-vertex $\frac{n}{2}$-bounded complete graph contains a properly edge-colored Hamiltonian cycle.*

In 2016, Lo [2] resolved the conjecture asymptotically.

Theorem 2 ([2]). *For any $\varepsilon > 0$, there is an integer n_0 such that if $n \geq n_0$, then every n-vertex $\left(\frac{n}{2} - \varepsilon\right)$-bounded complete graph contains a properly edge-colored Hamiltonian cycle.*

Recently, Montgomery announced that together with Milojević, Pokrovskiy and Sudakov, they solved the conjecture for large n.

Problem 3. *Is it true that every $\frac{\delta(G)}{2}$-bounded Dirac graph G contains a properly edge-colored Hamiltonian cycle?*

References

[1] B. Bollobás and P. Erdős. Alternating Hamiltonian cycles. *Israel J. Math.* , 23:126-131, 1976.

[2] A. Lo. Properly coloured Hamiltonian cycles in edge-coloured complete graphs. *Combinatorica*, 36(4):471-492, 2016.

The multiplicity of spectral radius of uniform hypergraphs

Proposed by Yi-Zheng Fan, Anhui University

Let $\mathcal{A} = (a_{i_1 \dots i_k})$ be a tensor of order k and dimension n with entries $a_{i_1 \dots i_k} \in \mathbb{C}$, where $i_j \in [n] := \{1, \dots, n\}$ and $j \in [k]$. Given a vector $x \in \mathbb{C}^n$, $\mathcal{A}x^{k-1} \in \mathbb{C}^n$, which is defined by $(\mathcal{A}x^{k-1})_i = \sum_{i_2,\dots,i_k \in [n]} a_{i i_2 \dots i_k} x_{i_2} \cdots x_{i_k}, i \in [n]$. For some $\lambda \in \mathbb{C}$, if the polynomial system $(\lambda\mathcal{I} - \mathcal{A})x^{k-1} = 0$ has a nontrivial solution x, then λ is an *eigenvalue* of $\mathcal{A}$ and x is an *eigenvector* of $\mathcal{A}$ associated with λ, where $\mathcal{I}$ is the identity tensor. The *characteristic polynomial* $\varphi_{\mathcal{A}}(\lambda)$ of $\mathcal{A}$ is defined as the resultant of the polynomials $(\lambda\mathcal{I} - \mathcal{A})x^{k-1}$, and λ is an eigenvalue of $\mathcal{A}$ if and only if it is a root of $\varphi_{\mathcal{A}}(\lambda)$. The *algebraic multiplicity* of λ is defined as the multiplicity of λ as a root of $\varphi_{\mathcal{A}}(\lambda)$, denoted by $\mathrm{am}(\lambda)$. The *spectral radius* of $\mathcal{A}$ is defined to be the largest modulus of the eigenvalues of $\mathcal{A}$.

The *eigenvariety* of $\mathcal{A}$ associated with an eigenvalue λ of $\mathcal{A}$ is defined to be the affine variety $\mathcal{V}_\lambda(\mathcal{A}) = \{x \in \mathbb{C}^n : \mathcal{A}x^{k-1} = \lambda x^{[k-1]}\}$. The *geometric multiplicity* of λ is defined as the dimension of $\mathcal{V}_\lambda(\mathcal{A})$, which is the maximum dimension of the irreducible components of $\mathcal{V}_\lambda(\mathcal{A})$, denoted by $\mathrm{gm}(\lambda)$. The *projective eigenvariety* of $\mathcal{A}$ associated with λ is defined to be the projective variety $\mathbb{V}_\lambda(\mathcal{A}) = \{x \in \mathbb{P}^{n-1} : \mathcal{A}x^{k-1} = \lambda x^{[k-1]}\}$, in the complex projective spaces $\mathbb{P}^{n-1}$ of dimension $n-1$. In 2016 Hu and Ye [3] proposed the following conjecture and showed that it is true in several cases.

Conjecture 1 ([3] Hu-Ye's conjecture)**.** *Suppose that a k-th order n-dimensional tensor $\mathcal{A}$ has an eigenvalue λ with eigenvariety $\mathcal{V}_\lambda(\mathcal{A})$ possessing κ irreducible components $V_1, \dots, V_\kappa$. Then*

$$\mathrm{am}(\lambda) \geq \sum_{i=1}^{\kappa} \dim(V_i)(k-1)^{\dim(V_i)-1}. \tag{1}$$

In particular,

$$\mathrm{am}(\lambda) \geq \mathrm{gm}(\lambda)(k-1)^{\mathrm{gm}(\lambda)-1}. \tag{2}$$

Cooper and Fickes [1] confirmed (1) in Hu-Ye's conjecture for the zero eigenvalue of $P_n^{(3)}$. Zheng [4] confirmed (2) in Hu-Ye's conjecture for all nonzero eigenvalues of a hyperpath $P_n^{(k)}$, and all nonzero eigenvalues of a hyperstar $S_n^{(k)}$. We have proved the following result.

Theorem 2 ([2])**.** *If H is one of the following hypergraphs: k-uniform hypertrees, k-th power of connected simple graphs, complete 3-uniform hypergraphs on at least 4 vertices, then for any eigenvalue λ of the adjacency tensor $\mathcal{A}(H)$ of H with modulus equal to the spectral radius,*

$$\mathrm{am}(\lambda) = |\mathbb{V}_\lambda(\mathcal{A}(H))|.$$

Consequently, (1) *in Hu-Ye's conjecture holds equality for the eigenvalue λ of $\mathcal{A}(H)$.*

We pose the following conjecture.

Conjecture 3. *Let H be a connected k-uniform hypergraph with spectral radius ρ. Then*

$$\mathrm{am}(\rho) = |\mathbb{V}_\rho(\mathcal{A}(H))|.$$

References

[1] J. Cooper, G. Fickes, Geometrics vs algebraic nullity for hyperpaths, *Pure Appl. Math. Q.*, 18(2022), 2433-2460.

[2] Yi-Zheng Fan, The multiplicity of eigenvalues of nonnegative tensors and uniform hypergraphs, arXiv: 2410.20830.

[3] S. Hu, K. Ye, Multiplicities of tensor eigenvalues, *Commun. Math. Sci.*, 14(2016), 1049-1071.

[4] Y.-N. Zheng, Algebraic and geometric multiplicities for some hypergraphs, *Discrete Math.*, 347(2024), 113780.

Kohayakawa–Nagle–Rödl–Schacht Conjecture for Chordal Graphs

Proposed by Ruonan Li, Northwestern Polytechnical University

Definition 1 (Locally dense graph)**.** *A graph G is (ε, p)-dense if for every $U \subseteq V(G)$ with $|U| \geq \varepsilon|V(G)|$, there holds $2|E(G[U])| \geq p|U|^2$.*

Conjecture 2 (KNRS Conjecture [1], JCTB, 2010))**.** *Let H be a graph. Then for every $p, \eta \in (0,1)$, there exists $\varepsilon = \varepsilon(p, \eta, H)$ and $n_0 = n_0(p, \eta, H)$ such that if G is (ε, p)-dense with $|V(G)| = n \geq n_0$, then*

$$\hom(H, G) \geq (1 - \eta)p^{|E(H)|}n^{|V(H)|}.$$

Joonkyung Lee [2] asked whether the KNRS conjecture holds for chordal graphs.

Question 3. *The conjecture holds for cliques. Can we prove it for chordal graphs? Specifically for each chordal graph with clique number 3?*

References

[1] Y. Kohayakawa, B. Nagle, V. Rödl and M. Schacht, Weak hypergraph regularity and linear hypergraphs, *Journal of Combinatorial Theory. Series B*, 2010, 100(2): 151–160.

[2] J. Lee, On some graph densities in locally dense graphs, *Random Structures & Algorithms* **58(2)** (2021) 322–344.

Some spectral extremal problems on books

Proposed by Yongtao Li, Tsinghua University

The famous Mantel theorem asserts that every n-vertex graph with at least $\lfloor n^2/4 \rfloor + 1$ edges contains a triangle. The bound is the best possible when we take G as a balanced complete bipartite graph $T_{n,2}$. A book of size t is a graph that consists of t triangles sharing a common edge. Edwards (unpublished), and independently by Khadžiivanov and Nikiforov (unavailable) proved that if G is an n-vertex graph with $e(G) \geq \lfloor n^2/4 \rfloor + 1$, then G contains a book of size greater than $n/6$, and this bound is tight. This solves a conjecture of Erdős; see [1, 2] for two alternative proofs.

The *adjacency matrix* of an n-vertex graph G is defined as $A(G) = [a_{ij}]_{i,j=1}^n$, where $a_{ij=1}$ if $ij \in E(G)$, and $a_{ij} = 0$ otherwise. The *spectral radius* $\lambda(G)$ of G is defined as the maximum modulus of eigenvalues of $A(G)$. By the Perron–Frobenius theorem, we know that $\lambda(G)$ is actually a largest eigenvalue of $A(G)$. Zhai and Lin [3, Problem 1.2] proposed the following interesting problem:

Problem 1. *For arbitrary positive integer n, if G is a graph on n vertices with $\lambda(G) > \lambda(T_{n,2})$, is it true that G contains a book of size greater than $\frac{n}{6}$?*

The above spectral problem is stronger than the aforementioned edge version, since $e(G) > e(T_{n,2})$ implies $\lambda(G) > \lambda(T_{n,2})$. Zhai and Lin [3] proved that such a graph G has a book of size $\frac{n}{6.5}$.

An old result of Nosal says that if G is an m-edge graph with $\lambda(G) > \sqrt{m}$, then G contains a triangle. Recently, Li, Liu and Zhang [4, Problem 6.1] studied the spectral extremal problem for graphs with given number of edges, and proposed the following problem.

Problem 2. *Does every m-edge graph G with $\lambda(G) > \sqrt{m}$ contains a book of size $\frac{1}{3}\sqrt{m}$?*

For related spectral results on counting substructures, we refer to [5, 6].

References

[1] Béla Bollobás, Vladimir Nikiforov, Books in graphs, *European J. Combin.* **26** (2005) 259–270.

[2] Yongtao Li, Lihua Feng, Yuejian Peng, A spectral Erdős–Faudree–Rousseau theorem, *J. Graph Theory* **110** (2025) 408–425.

[3] Mingqing Zhai, Huiqiu Lin, A strengthening of the spectral color critical edge theorem: Books and theta graphs, *J. Graph Theory* **102** (3) (2023) 502–520.

[4] Yongtao Li, Hong Liu, Shengtong Zhang, More on Nosal's spectral theorem: Books and 4-cycles, 23 pages, (2025), arXiv:2508.14366.

[5] Bo Ning, Mingqing Zhai, Counting substructures and eigenvalues I: Triangles, *European J. Combin.* **110** (2023), No. 103685.

[6] Bo Ning, Mingqing Zhai, Counting substructures and eigenvalues II: Quadrilaterals, *Electron. J. Combin.* **32** (4) (2025), #P4.1

A problem of Erdős on $r(C_4, K_n)$

Proposed by Qizhong Lin, Fuzhou University

It is known that there exist constants $c_1, c_2 > 0$ such that for all sufficiently large n,

$$c_1 \left(\frac{n}{\log n}\right)^{3/2} \leq r(C_4, K_n) \leq c_2 \left(\frac{n}{\log n}\right)^2, \tag{3}$$

where the lower bound is proved by probabilistic methods [4], and the upper bound is due to Szemerédi (unpublished, see [2]).

Conjecture 1 (Erdős [1]). *there exists a constant $\varepsilon > 0$ such that*

$$r(C_4, K_n) = o(n^{2-\varepsilon}).$$

We give a short proof the upper bound of (3) using the following theorem. For any vertex v of G, let G_v be the subgraph induced by the neighborhood of v.

Theorem 2 (Li and Rousseau [3]). *Let $a \geq 0$ be an integer. Let G be a graph with N vertices and average degree d. For any vertex v of G, if G_v has maximum degree at most a, then*

$$\alpha(G) \geq N f_{a+1}(d) \geq N \frac{\log(d/(a+1)) - 1}{d}.$$

Proof of the upper bound of (3). Let G be a graph of order $N = r(C_4, K_n) - 1$ which contains no C_4 and $\alpha(G) < n$. For any vertex v of G, the maximum degree of G_v is at most 1 since G does not contain C_4. Moreover, since $ex(N, C_4) \leq (1/2 + o(1))N^{3/2}$, we have that the average degree of G is at most $(1 + o(1))N^{1/2}$. We apply Theorem 2 with $a = 1$ to obtain that

$$n > \alpha(G) \geq (1 - o(1))N \frac{\log N^{1/2}}{N^{1/2}} > \frac{1}{2} N^{1/2} \log N.$$

The upper bound follows. □

References

[1] P. Erdős, Extremal problems in number theory, combinatorics and geometry, in *Proc. of the International Congress of Mathematicians*, Vol. 1, 2 (Warsaw, 1983), 51–70. Warsaw: PWN, 1984.

[2] P. Erdős, R. J. Faudree, C. C. Rousseau, and R. H. Schelp, On cycle-complete graph Ramsey numbers, *J. Graph Theory* 2 (1978), 53–64.

[3] Y. Li and C. C. Rousseau, On book-complete graph Ramsey numbers, *J. Combin. Theory Ser. B* 68 (1996), 36–44.

[4] J. Spencer, Asymptotic lower bounds for Ramsey functions, *Discrete Math.* 20 (1977/78), 69–76.

Extremal Spectral Radius Problem

Proposed by Zhenzhen Lou, University of Shanghai for Sceince and Technology

Problem 1 (Brualdi, Solheid, [1]). *Given a set* $\mathbb{G}$ *of graphs, find* $\min\{\rho(G) : G \in \mathbb{G}\}$ *and* $\max\{\rho(G) : G \in \mathbb{G}\}$*, and characterize the graphs which achieve the minimum or maximum value.*

In general, characterizing the graphs of the minimum spectral radius is still an open problem [2]. Particularly, Stevanović [2] pointed out that determining the graph with the minimum spectral radius among connected graph with independence number α appears to be a tough problem.

Problem 2 ([2]). *What is the minimal spectral radii of graphs with given independence number* α*?*

Define

$$\rho_D(n) := \min\bigl\{\rho(\Gamma) \bigm| \Gamma \text{ is a graph with } n \text{ vertices and diameter } D\bigr\}.$$

Conjecture 3 ([3]). *Let* $D \geq 1$*. Then*

$$\lim_{n\to\infty} \frac{\rho_D(n)}{\sqrt[D]{n-1}} = 1.$$

References

[1] R.A. Brualdi, E.S. Solheid, On the spectral radius of complementary acyclic matrices of zeros and ones, SIAM J. Algebra Discrete Method 7 (1986) 265–272.

[2] D. Stevanović, Spectral Radius of Graphs, Academic Press, Amsterdam, 2015.

[3] S. M. Cioabă, E. R. van Dam, J. H. Koolen, and J. H. Lee, A lower bound for the spectral radius of graphs with fixed diameter, European J. Combin., 31 (2010) 1560–1566.

The redundant subgraphs in the k-connected graphs

Proposed by Yingzhi Tian, Xinjiang University

For a k-connected G, a vertex subset or a subgraph R of G is said to be *redundant* if $G - R$ is still k-connected. In 1972, Chartrand, Kaigars and Lick [1] proved that there is a redundant vertex in a k-connected graph with minimum degree at least $\lfloor \frac{3k}{2} \rfloor$. In 2008, Fujita and Kawarabayashi [2] conjectured that every k-connected graph with $\delta(G) \geq \lfloor \frac{3k}{2} \rfloor + f_k(m) - 1$ contains a connected subgraph G' of order m such that $\kappa(G - V(G')) \geq k$, where $f_k(m)$ is nonnegative. Mader [4] confirmed this conjecture and proved that $f_k(m) = m$. In addition, the connected subgraph G' can be chosen as a path. Meanwhile, Mader conjectured that the result would hold even if the path is replaced by any tree with the same order.

Conjecture 1. *(Mader [4]) For any tree T with order m, every k-connected graph G with $\delta(G) \geq \lfloor \frac{3k}{2} \rfloor + m - 1$ contains a tree $T' \cong T$ such that $\kappa(G - V(T')) \geq k$.*

When G is a bipartite graph, Luo, Tian and Wu [3] showed that the minimum degree condition in the conjecture above can be relaxed to $k + m$ and proposed the bipartite version conjecture.

Conjecture 2. *([3]) For any tree T with bipartition X and Y, every k-connected bipartite graph G with minimum degree at least $k + t$, where $t = max\{|X|, |Y|\}$, contains a tree $T' \cong T$ such that $\kappa(G - V(T')) \geq k$.*

For more results, see the survey paper [5].

References

[1] G. Chartrand, A. Kaigars, D.R. Lick, Critically n-connected graphs, Proc. Amer. Math. Soc. 32 (1972) 63-68.

[2] S. Fujita, K. Kawarabayashi, Connectivity keeping edges in graphs with large minimum degree, J. Combin. Theory Ser. B 98 (2008) 805-811.

[3] L. Luo, Y. Tian, L. Wu, Connectivity keeping paths in k-connected bipartite graphs, Discrete Math. 345 (4) (2022) 112788.

[4] W. Mader, Connectivity keeping paths in k-connected graphs, J. Graph Theory 65 (2010) 61-69.

[5] Y. Tian and J. Meng, A survey on the vertex-(edge-)k-maximal graphs and the k-vertex- (edge-)connected graphs with redundant subgraphs, Discrete Appl. Math. 378 (2026) 125–135.

Intersecting k-graphs with covering number k

Proposed by Jian Wang, Taiyuan University of Technology

Let $[n]$ denote the standard set $\{1, 2, \ldots, n\}$ and let $\binom{[n]}{k}$ denote the family of all k-element subsets of $[n]$. A subfamily of $\binom{[n]}{k}$ is called a *k-graph*. We say that $\mathcal{F} \subset \binom{[n]}{k}$ is *intersecting* if $F \cap F' \neq \emptyset$ for all $F, F' \in \mathcal{F}$.

One of the most important results in extremal set theory is the Erdős-Ko-Rado Theorem.

Erdős-Ko-Rado Theorem ([3]). Let $n \geq 2k$ and suppose that $\mathcal{F} \subset \binom{[n]}{k}$ is intersecting then

$$|\mathcal{F}| \leq \binom{n-1}{k-1}. \tag{4}$$

For $\mathcal{F} \subset \binom{[n]}{k}$, define the *covering number* $\tau(\mathcal{F})$ as the minimum size of T that satisfies $F \cap T \neq \emptyset$ for all $F \in \mathcal{F}$. In their seminal paper [4], Erdős and Lovász (among other things) investigated

$$m(k) = \max\left\{|\mathcal{F}| \colon \mathcal{F} \subset \binom{[n]}{k} \text{ is intersecting, } n \text{ is arbitrarily large, } \tau(\mathcal{F}) = k\right\}.$$

Erdős-Lovász Theorem ([4]).

$$\lfloor (e-1)k! \rfloor \leq m(k) \leq k^k. \tag{5}$$

In [8] Lovász conjectured that $m(k)$ equals the lower bound of (5). However it was disproved in [6] for $k = 4$. The constructions in [7] show that

$$m(k) \geq (1 + o(1)) \left(\frac{k}{2}\right)^k. \tag{6}$$

The upper bound part of (5) was improved sequentially in [9], [2], [1], [5], [10].

Theorem 1 ([10])**.**

$$m(k) \leq e^{-k^{0.5+o(1)}} k^k. \tag{7}$$

Conjecture 2 (Frankl)**.** *There exists an $\varepsilon > 0$ such that for $n > n_0(k)$ and $k > k_0$,*

$$m(k) \leq ((1-\varepsilon)k)^k.$$

References

[1] A. Arman, R. Troy, An upper bound for the size of a k-uniform intersecting family with covering number k, J. Comb. Theory, Ser. A 147 (2017), 18–26.

[2] D. Cherkashin, About maximal number of edges in hypergraph-clique with chromatic number 3, arXiv:1107.1869 (2011).

[3] P. Erdős, C. Ko, R. Rado, Intersection theorems for systems of finite sets, Quart. J. Math. Oxford Ser. 12 (1961), 313–320.

[4] P. Erdős, L. Lovász, Problems and results on 3-chromatic hypergraphs and some related questions, in "Infinite and Finite Sets," Proc. Colloquium Math. Society Janos Bolyai, Vol. 10, (A. Hajnal et al., Eds.), pp. 609–627, North-Holland, Amsterdam, 1975.

[5] P. Frankl, A near-exponential improvement of a bound of Erdős and Lovász on maximal intersecting families, Combinatorics, Probability & Computing 28.5 (2019), 733–739.

[6] P. Frankl, K. Ota, N. Tokushige, Uniform intersecting families with covering number four, J. Comb. Theory, Ser. A 71 (1) (1995), 127–145.

[7] P. Frankl, K. Ota, N. Tokushige, Covers in uniform intersecting families and a counterexample to a conjecture of Lovász, J. Comb. Theory, Ser. A 74(1) (1996), 33–42.

[8] L. Lovász, On minimax theorems of combinatorics, Math. Lapok 26 (1975), 209–264.

[9] Z. Tuza, Inequalities for minimal covering sets in set systems of given rank, Discrete Applied Mathematics 51 (1994), 187–195

[10] D. Zakharov, On the size of maximal intersecting families, Combinatorics, Probability &Computing 33 (2024), 32–49.

Graham's GCD conjecture and its connections with Farey sequence and graph theory

Proposed by Liuquan Wang, Wuhan University

In 1970, Graham [3] proposed the following conjecture.

Conjecture 1. *Let $a_1, a_2, \cdots, a_n$ be distinct positive integers, we have*

$$\max_{i,j} \frac{a_i}{(a_i, a_j)} \geq n.$$

Balasubramanian and Soundararajan [1] confirmed this conjecture based on deep analytical methods.

Given a positive integer n, the Farey sequence F_n is the set of rational numbers a/b with $0 \leq a \leq b \leq n$ and $(a,b) = 1$. For any set S of real numbers, we define

$$\mathcal{Q}(S) = \left\{ \frac{x}{y} : x, y \in S, x \leq y \text{ and } y \neq 0 \right\}.$$

In particular, if $S = \{0\}$ then we agree that $\mathcal{Q}(S) = \{0\}$. We [4] find the following theorem which is equivalent to Conjecture 1.

Theorem 2. *Suppose $S \subseteq F_n$, if $\mathcal{Q}(S) \subseteq F_n$, then S has at most $n+1$ elements.*

Problem 3. *Can we prove Theorem 2 directly and thus providing new proofs for Graham's conjectures?*

Let k be a positive integer. Bosek, Debski, Grytczuk, Sokól, Śleszyńska-Nowak and Zelazny [2] defined auxiliary graphs reflecting in some sense the arithmetic proximity of numbers. Define arithmetic proximity between two integers a and b as

$$AP(a,b) := \max\{\frac{a}{(a,b)}, \frac{b}{(a,b)}\}.$$

Two numbers are arithmetically close if the above value is relatively small.

Let k be a fixed positive integer. Define an infinite graph B_k on the set $\mathbb{N}$ by joining a to b if and only if their arithmetic proximity is at most k. These graphs are called as *arithmetic graphs*. Bosek et al. [2] considered the following

Problem 4. *What is the chromatic number (denoted as $\chi(B_k)$) of B_k?*

Clearly the numbers $1, \ldots, k$ form a clique, so there is no hope for coloring that would use less than k colors. Hence $\chi(B_k) \geq k$. The following conjecture is stronger than Conjecture 1.

Conjecture 5 (Bosek et al. [2])**.** *Every arithmetic graph B_k satisfies $\chi(B_k) = k$.*

In this talk, we will present some facts about Conjecture 5 and some interesting applications [4] of Graham's conjecture to Farey sequences.

References

[1] R. Balasubramanian and K. Soundararajan, On a conjecture of R. L. Graham, Acta Arith. 75 (1996), 1–38.

[2] B. Bosek, M. Debski, J. Grytczuk, J. Sokól, M. Śleszyńska-Nowak, W. Zelazny, Graph coloring and Graham's greatest common divisor problem, Disc. Math. 341 (2018), 781–785.

[3] R.L. Graham, Unsolved problem 5749, Amer. Math. Monthly, 77 (1970), 775.

[4] L. Wang, Farey sequence and Graham's conjecture, J. Number Theory 229 (2021), 399–404.

Packing colorings of graphs

Proposed by Yan Wang, Shanghai Jiao Tong University

An i-packing in a graph G is a set of vertices whose pairwise vertex distance is at least $i+1$. Let $S = (s_1, s_2, ..., s_k)$ be a non-decreasing sequence of positive integers. A *packing S-coloring* of a graph G is a partition of $V(G)$ into sets $V_1, ..., V_k$ such that each V_i is an s_i-packing.

Gastineau and Togni [1] proved that every subcubic graph is packing $(1^2, 2^3)$-colorable. Furthermore, they asked

Problem 1 (Gastineau and Togni [1]). *Is it true that every subcubic graph except the Petersen graph is packing* $(1, 1, 2, 3)$*-colorable?*

This question still remains open. Very recently, Liu, Zhang and Zhang [3] showed every subcubic graph is packing $(1, 1, 2, 2, 3)$-colorable and conjectured that every subcubic graph except the Petersen graph is packing $(1^2, 2^2)$-colorable.

For packing $(1, 2^k)$-coloring, Tarhini and Togni [4] proved that every cubic Halin graph is packing $(1, 2^5)$-colorable. Gastineau and Togni [1] proved every subcubic graph is packing $(1, 2^6)$-colorable. Liu and Wang [2] showed that every subcubic planar graph is packing $(1, 2^5)$-colorable. Gastineau and Togni [1] asked the following question after performing a computer search on graphs with small order.

Problem 2 (Gastineau and Togni [1]). *Is it true that every subcubic graph except the Petersen graph is packing* $(1, 2^5)$*-colorable?*

References

[1] N. Gastineau and O. Togni, S-packing colorings of cubic graphs, *Discrete Math.* **339** (2016), 2461–2470.

[2] X. Liu and Y. Wang, Partition subcubic planar graphs into independent sets, arXiv:2408.12189.

[3] X. Liu, X. Zhang and Y. Zhang, Every subcubic graph is packing $(1, 1, 2, 2, 3)$-colorable, *Discrete Math.* **348** (11) (2025), 114610.

[4] B. Tarhini and O. Togni, S-Packing Coloring of Cubic Halin Graphs, *Discrete Appl. Math.* **349** (31) (2024), 53–58.

The frustration index of a signed graph

Proposed by Zhouningxin Wang, Nankai University

A signed graph (G, σ) consists of an underlying graph G together with a signature $\sigma : E(G) \to \{+1, -1\}$. Let $E^-_{(G,\sigma)}$ denote the set of negative edges in (G, σ). The operation of *switching* at a vertex is to multiply the signs of all incident edges by -1. Two signatures σ and π on a graph G are said to be *switching-equivalent* if one can be obtained from the other by a sequence of switching operations at some vertices. The *frustration index* of a signed graph (G, σ), denoted by $F(G, \sigma)$, is defined as

$$F(G, \sigma) = \min\{|E^-_{(G,\sigma)}| \mid \sigma' \text{ is switching-equivalent to } \sigma\}.$$

Equivalently, the frustration index is the minimum number of edges that need to be removed so that the resulting signed graph can be switched into an all-positive one; this quantity is also known as the minimum negative-cycle covering number.

One nice result obtained in [1] is the following: If a signed graph is odd-K_4-minor-free or Eulerian odd-K_5-minor-free, then its frustration index equals to the maximum number of edge disjoint negative cycles in it.

Problem 1. *Charaterize the class of signed graphs (G, σ) such that its frustration index is equal to to the maximum number of edge disjoint negative cycles.*

Another interesting class of signed graphs is that of special signed hypercubes. Let H_n denote the n-dimensional hypercube, and let $\sigma^* : E(H_n) \to \{+1, -1\}$ be a signature such that every 4-cycle is negative. A classical question posed by P. Erdős asks for the minimum number $f(n)$ of edges that must be removed from H_n in order to make the resulting graph C_4-free. The following question is in a similar spirit.

Problem 2. *Determine $F(H_n, \sigma^*)$.*

It is known that $(n - \sqrt{n}) \cdot 2^{n-2} \leq F(H_n, \sigma^*) \leq 2^{n-2} \left(n - \frac{(n+2\cdot 4^k - \frac{m}{2})}{3\cdot 2^k} \right)$ where $4^k \leq n < 4^{k+1}$ and $n + m = 1 \pmod 3$ with $m \in \{0, 1, 2\}$. Out of personal curiosity, one may ask whether $f(n) = F(H_n, \sigma^*)$.

References

[1] James F. Geelen, and Bertrand Guenin, Packing Odd Circuits in Eulerian Graphs, *Journal of Combinatorial Theory, Series B*, Volume 86, Issue 2, 2002, Pages 280-295.

Subdivisions with length constraints

Proposed by Donglei Yang, Shandong University

For a graph H, a *subdivision* of H, denoted by TH, is a graph obtained by replacing edges of H by internally vertex-disjoint paths. As a fundamental question, the extremal problems on H-subdivision have been extensively studied, starting from a result of Mader from 1967. He showed that for every $k \in \mathbb{N}$, there exists (finite) $f(k)$ such that every graph G with average degree at least $f(k)$ contains a TK_k. Mader furthermore conjectured that one can take $f(k) = O(k^2)$. This conjecture was finally resolved in the 90s by Bollobás and Thomason and independently by Komlós and Szemerédi.

Recent trends have been focusing on the existence of subdivisions with length constraints. An old question of Erdős asks for every $\varepsilon > 0$, whether there exists $\delta > 0$ such that every graph with n vertices and average degree at least εn contains a $K_{\delta\sqrt{n}}$-subdivision where each edge is subdivided exactly once. Alon, Krivelevich and Sudakov affirmatively answered the question with $\delta = \varepsilon^{\frac{3}{2}}$, and this result was improved to $\delta = \varepsilon$ by Fox and Sudakov. In this direction, Dvořák asked (in his thesis) whether one can strengthen the above-mentioned result of Bollobás–Thomason and Komlós–Szemerédi for polynomially dense graphs.

Conjecture 1 (Dvořák)**.** *For given $\alpha > 0$, every graph G with $d(G) = d \geq n^{\alpha}$ contains a $TK_{\Omega(\sqrt{d})}$, where each edge is subdivided $O_{\alpha}(1)$ times.*

A very recent result of Tomon[1] implies

(a) an ℓ-subdivision of K_t with $\ell = O(\frac{1}{\alpha}\log\frac{1}{\alpha})$ and $t = n^{\Theta(\alpha)}$.

(b) an ℓ-subdivision of K_t with $\ell = O(\frac{1}{\varepsilon}\log\frac{1}{\varepsilon})$ and $t = \frac{\sqrt{d}}{n^{\varepsilon}}$.

Tomon conjectured that $\ell = O(\frac{1}{\alpha}\log\frac{1}{\alpha})$ in (a) can be replaced by $\ell = O(\frac{1}{\alpha})$.

References

[1] I. Tomon, Robust (rainbow) subdivisions and simplicial cycle, Advances in Combinatorics, 2024, http://dx.doi.org/10.19086/aic.2024.1.

Open problems of divisible subdivisions

Proposed by Fan Yang, Shandong University

A K_f*-minor* is defined as a graph G whose vertex set is partitioned into f disjoint non-empty sets $X_1, \ldots, X_f$, such that for every $i \in [f]$, the induced subgraph $G[X_i]$ is connected, and for every $i \neq j \in [f]$, there exists at least one edge in G with endpoints in X_i and X_j. Given a graph H, a *subdivision* of H is any graph H' obtained from H by replacing its edges with internally vertex-disjoint paths connecting the original endpoints of the edges in H. The paths in H' replacing the edges of H are called *subdivision paths.* A subdivision H' of H is called q*-divisible* if all its subdivision paths are of length divisible by q.

Theorem 1 ([1]). *For every graph H with $\Delta(G) \leq 3$ and every integer $q \geq 2$ there exists a (smallest) integer $f = f(H, q) \geq 1$ such that every K_f-minor contains a q-divisible subdivision of H as a subgraph.*

Remark. Through the proof of Theorem 1, it gives an upper bound on $f(H, q)$ which is of magnitude $(q^2 n)^{q^3 n}$, where $n = |V(H)|$. However, the best lower bound on $f(H, q)$ for subcubic graphs H on n vertices and m edges is $m(q-1) + n$, obtained by considering the complete graph of order $m(q-1) + n - 1$ (which is too small to host a q-divisible subdivision of H).

Alon and Krivelevich posed the problem of improving their superexponential bound on $f(H, q)$.

Theorem 2 ([2]). *Let H be an n-vertex graph with $e(H) = m$ and $\Delta(H) \leq 3$. Then, for every integer $q \geq 2$, it holds that*

$$m(q-1) + n \leq f(H, q) \leq 7mq + 8n + 14q,$$

and hence $f(H, q) = \Theta(mq + n)$.

Problem 3 ([2]). *Given $q \in \mathbb{N}$ and a subcubic graph H with n vertices and m edges, is it true for $f = m(q-1) + n$ that every subdivision of K_f contains a q-divisible H-subdivision?*

Problem 3 is true when $q = 2$ and $H = K_4$.

References

[1] N. Alon, M. Krivelevich, Divisible subdivisions, Journal of Graph Theory 98 (2021) 623-629.

[2] S. Das, N. Draganić, R. Steiner, Tight bounds for divisible subdivisions, Journal of Combinatorial Theory, Series B, 165 (2024) 1-19.

Expander problem

Proposed by Tao Zhang, Xidian University

Let $A \subset \mathbb{F}_q$, and define

$$(A-A)(A-A) = \{(a-b)(c-d) : a,b,c,d \in A\}.$$

Let $\alpha \in [0,1]$ denote the smallest exponent such that, for any $A \subset \mathbb{F}_q$ with $|A| \geq Cq^{\alpha}$ (for sufficiently large C), we have

$$(A-A)(A-A) = \mathbb{F}_q.$$

It follows from [1, 2] that

$$\frac{2}{3} \leq \alpha \leq \frac{3}{4}.$$

Conjecture 1. $\alpha = \frac{3}{4}$.

References

[1] A. Iosevich and M. Rudnev. Erdős distance problem in vector spaces over finite fields. *Trans. Amer. Math. Soc.*, 359(12):6127–6142, 2007.

[2] B. Murphy and G. Petridis. Products of difference over arbitrary finite fields. *Discrete Anal.*, pages Paper No. 18, 42, 2018.

List colouring of Eulerian triangulations

Proposed by Xuding Zhu, Zhejiang Normal University

An Eulerian triangulation of the plane is a triangulation of the plane in which each vertex has even degree. It is known that Eulerian triangulations of the plane are 3-colourable. Mirzakhani constructed an Eulerian triangulation of the plane which is not 4-choosable.

Assume $\lambda = \{k_1, k_2, \ldots, k_q\}$ is a multiset of positive integers. Let $k_\lambda = \sum_{i=1}^{q} k_i$. So λ is a partition of the integer k_λ. A λ-assignment of a graph G is a k_λ-assignment L such that the set of colours

$$C = \cup_{v \in V(G)} L(v)$$

can be partitioned into q subsets,

$$C = C_1 \cup C_2 \cup \ldots \cup C_q$$

and for each vertex v,

$$|L(v) \cap C_i| = k_i.$$

We say G is λ-choosable if G is L-colourable for each λ-assignment L.

Question 1. *Is it true that every Eulerian triangulation G of the plane is $\{2, 2\}$-choosable?*

Question 2. *Is it true that for any Eulerian triangulation G of the plane, for any 2-assignment L of G, there is an L-colouring ϕ of G such that for each colour c, $\phi^{-1}(c)$ induces a bipartite graph?*

A positive answer to Question 2 is a weakening of the following conjecture of Kundergen et al.

Conjecture 3. *For any planar graph G, for any 2-assignment L of G, there is an L-colouring ϕ of G such that for each colour c, $\phi^{-1}(c)$ induces a bipartite graph.*

Conjecture 3 is stronger than the Four Colour Theorem.

The following is a strengthening of Question 2 as well as the well-known Barnette's conjecture:

Question 4. *Is it true that for any Eulerian triangulation G of the plane, for any 2-assignment L of G, there is an L-colouring ϕ of G such that for each colour c, $\phi^{-1}(c)$ induces a tree ?*

Barnette's conjecture is the non-list version of Question 4 (or all the lists consists of the same two colours).

Registered participants

(1) **Jiangdong Ai**, *Nankai University*
jd@nankai.edu.cn

(2) **Yandong Bai**, *Northwestern Polytechnical University*
bai@nwpu.edu.cn

(3) **Ilkyoo Choi**, *Hankuk University of Foreign Studies*
ilkyoochoi@gmail.com

(4) **Kecai Deng**, *Huaqiao University*
kecaideng@126.com

(5) **Laihao Ding**, *Central China Normal University*
dinglaihao@ccnu.edu.cn

(6) **Yizheng Fan**, *Anhui University*
fanyz@ahu.edu.cn

(7) **Tao Feng**, *Zhejiang University*
tfeng@zju.edu.cn

(8) **Shinya Fujita**, *Yokohama City University*
shinya.fujita.ph.d@gmail.com

(9) **Changxiang He**, *University of Shanghai for Science and Technology*
changxianghe@hotmail.com

(10) **Xiaolan Hu**, *Central China Normal University*
xlhu@mail.ccnu.edu.cn

(11) **Ping Hu**, *Sun Yat-sen University*
huping9@mail.sysu.edu.cn

(12) **Xinqi Huang**, *University of Science and Technology of China & IBS ECOPRO*
huangxq@mail.ustc.edu.cn

(13) **Suyun Jiang**, *Jianghan University*
jiang.suyun@163.com

(14) **Masaki Kashima**, *Keio University*
masaki.kashima10@gmail.com

(15) **Seog-Jin Kim**, *Konkuk University*
seogjin@gmail.com

(16) **Younjin Kim**, *POSTECH*
mathyounjinkim@gmail.com

(17) **Ander Lamaison**, *IBS ECOPRO*
ander@ibs.re.kr

(18) **Joonkyung Lee**, *Yonsei University*
joonkyunglee@yonsei.ac.kr

(19) **Ruonan Li**, *Northwestern Polytechnical University*
rnli@nwpu.edu.cn

(20) **Binlong Li**, *Northwestern Polytechnical University*
binlongli@nwpu.edu.cn

(21) **Shuchao Li**, *Central China Normal University*
li@ccnu.edu.cn

(22) **Yongtao Li**, *Tsinghua University*
ytli0921@hnu.edu.cn

(23) **Weicong Li**, *Great Bay University*
liweicong@gbu.edu.cn

(24) **Yusheng Li**, *Tongji University*
li_yusheng@tongji.edu.cn

(25) **Qizhong Lin**, *Fuzhou University*
linqizhong@fzu.edu.cn

(26) **Huiqiu Lin**, *East China University of Science and Technology*
huiqiulin@ecust.edu.cn

(27) **Lele Liu**, *Anhui University*
liu@ahu.edu.cn

(28) **Hong Liu**, *IBS ECOPRO*
hongliu@ibs.re.kr

(29) **On-Hei Solomon Lo**, *Tongji University*
oslo@tongji.edu.cn

(30) **Zhenzhen Lou**, *University of Shanghai for Science and Technology*
louzz@usst.edu.cn

(31) **Hongliang Lu**, *Xi'an Jiaotong University*
luhongliang@mail.xjtu.edu.cn

(32) **You Lu**, *Northwestern Polytechnical University*
luyou@nwpu.edu.cn

(33) **Changhong Lv**, *East China Normal University*
chlu@math.ecnu.edu.cn

(34) **Shun-ichi Maezawa**, *Nihon University*
maezawa.mw@gmail.com

(35) **Yaping Mao**, *Qinghai Normal University*
yapingmao@outlook.com

(36) **Zhengke Miao**, *Jiangsu Normal University*
zkmiao@jsnu.edu.cn

(37) **Suil O**, *The State University of New York, Korea*
suil.oh@gmail.com

(38) **Boram Park**, *Seoul National University*
borampark@snu.ac.kr

(39) **Lihong Qiu**, *University of Shanghai for Science and Technology*
qiulihong@usst.edu.cn

(40) **Shohei Satake**, *Kumamoto University*
satakecomb@gmail.com

(41) **Bjarne Schülke**, *IBS ECOPRO*
bjarneschuelke@gmail.com

(42) **Haiying Shan**, *Tongji University*
shan_haiying@tongji.edu.cn

(43) **Chong Shangguan**, *Shandong University (Qingdao)*
theoreming@163.com

(44) **Yingzhi Tian**, *Xinjiang University*
tianyzhxj@163.com

(45) **Shoichi Tsuchiya**, *Senshu University*
s.tsuchiya@isc.senshu-u.ac.jp

(46) **Yi Wang**, *Anhui University*
wangy@ahu.edu.cn

(47) **Jian Wang**, *Taiyuan University of Technology*
wangjian01@tyut.edu.cn

(48) **Zhouningxin Wang**, *Nankai University*
wangzhou@nankai.edu.cn

(49) **Liuquan Wang**, *Wuhan University*
wanglq@whu.edu.cn

(50) **Yan Wang**, *Shanghai Jiao Tong University*
yan.w@sjtu.edu.cn

(51) **Jun Wang**, *Shanghai Normal University*
jwang@shnu.edu.cn

(52) **Hengjia Wei**, *Xi'an Jiaotong University*
hjwei05@xjtu.edu.cn

(53) **Meiqin Wei**, *Shanghai Maritime University*
weimeiqin8912@163.com

(54) **Tingzeng Wu**, *Qinghai Minzu University*
mathtzwu@163.com

(55) **Hehui Wu**, *Fudan University*
hhwu@fudan.edu.cn

(56) **Zixiang Xu**, *IBS ECOPRO*
zixiangxu@ibs.re.kr

(57) **Yujun Yang**, *Yantai University*
yangyj@ytu.edu.cn

(58) **Donglei Yang**, *Shandong University*
dlyang@sdu.edu.cn

(59) **Fan Yang**, *Shandong University*
fyang@sdu.edu.cn

(60) **Xiaowei Yu**, *Jiangsu Normal University*
xwyu@jsnu.edu.cn

(61) **Xingzhi Zhan**, *East China Normal University*
zhan@math.ecnu.edu.cn

(62) **Tao Zhang**, *Xidian University*
zhant220@163.com

(63) **Ping Zhang**, *University of Shanghai for Science and Technology*
mathzhangping@126.com

(64) **Qinghong Zhao**, *Huaqiao University*
qzhao@hqu.edu.cn

(65) **Xuding Zhu**, *Zhejiang Normal University*
xdzhu@zjnu.edu.cn

(66) **Yan Zhu**, *University of Shanghai for Science and Technology*
zhuyan@usst.edu.cn