Early Career Workshop in Extremal Combinatorics

Institute for Basic Science (IBS)

October 20–31, 2025

Organizing Committee
Felix Clemen (University of Victoria, Canada)
Joonkyung Lee (Yonsei University, Korea)
Hong Liu (IBS ECOPRO, Korea)
Bjarne Schülke (IBS ECOPRO, Korea)

Workshop Overview

The Early Career Workshop in Extremal Combinatorics aims to bring together young researchers to discuss problems, exchange ideas, and work collaboratively. The schedule is as follows:

- **October 20:** Presentations by Participants
- **October 21:** Problem Proposal Session
- **October 22–24:** Group Work on Problems
- **October 27–31:** Continuation of Group Work

Monday, October 20: Presentations by Participants

We will be meeting at 9:30 in the IBS lobby to get a visitor pass. The first day will feature 8 talks by participants. Each talk is allocated 30 minutes. The schedule is as follows:

Time	Topic	Speaker
10:00–10:30	The semi-inducibility problem	Bertille Granet
10:30–11:00	A story of circles and parabolas	Gabriel Currier
11:00–11:30	Independent sets in percolated graphs via the Ising model	Anna Geisler
11:30–12:00	On Kotzig's conjecture in random graphs	Amadeo Sgueglia
12:00–13:00	**Lunch Break**	
13:00–13:30	The maximum diameter of d-dimensional simplicial complexes	Silas Rathke
13:30–14:00	The Ramsey Multiplicity Problem	Jae-baek Lee
14:00–14:30	**Break**	
14:30–15:00	Double-jump phase transition for the reverse Littlewood–Offord problem	Victor Souza
15:00–15:30	Geometric hypergraphs	Gábor Damásdi

Tueday, October 21: Problem Proposal Session 1:

Start: 10am.

Abstracts of Talks

The abstracts for the talks on October 20 are provided below.

The semi-inducibility problem (Bertille Granet, Monash University)

Let H be a k-edge-coloured graph and let n be a positive integer. What is the maximum number of copies of H in a k-edge-coloured complete graph on n vertices? We study the case $k = 2$, which we call the semi-inducibility problem. This problem is a generalisation of the inducibility problem of Pippenger and Golumbic which is solved only for some small graphs and limited families of graphs. We prove sharp or almost sharp results for alternating walks, for alternating cycles of length divisible by 4, and for 4-cycles of every colour pattern.

Liu, Mubayi and Reiher asked whether there is a graph F for which the binomial random graph is an asymptotically extremal graph in the inducibility problem over all graphs of a given edge density. This was recently answered in a strong negative sense by Jain, Michelen and Wei. In contrast, we find a quantum graph Q with positive coefficients and an interval of edge densities for which the only extremal graphs are quasirandom.

This is joint work with Abdul Basit, Daniel Horsley, André Kündgen, and Katherine Staden.

A story of circles and parabolas (Gabriel Currier, UBC)

The Erdős unit distance problem asks the following: given n points in the plane, what is the maximum number of pairs of these points that can be at distance 1 apart? One can rephrase this question in terms of bounding the number of incidences between n points and n unit circles. One of the reasons this problem appears difficult is that it is not obvious which properties of the circle one might use to prove the hypothesized bound; in particular, there are curves that share many similar properties (in particular, unit parabolas) which appear to have very different behavior. In this talk, we discuss one way that circles and parabolas are different, and how we can leverage this to prove new results about unit distances when our direction set is relatively small. This is joint work with Jozsef Solymosi.

Independent sets in percolated graphs via the Ising model (Anna Geisler, TU Graz)

Given a graph G, we form a random subgraph G_p by including each edge independently with probability p. For bipartite, regular graphs G with certain expansion properties, we determine the expected number of independent sets in a random subgraph G_p. While in the deterministic setting independent sets are typically studied via the hard-core model, in the randomized setting we use a connection to the Ising model. Using the cluster expansion method from statistical physics, we obtain an expansion of the Ising model partition function within a suitable range of parameters. As a tool, we establish a refined container lemma for the Ising model, which yields a slight improvement over recent bounds of Jenssen, Malekshahian, and Park for the hard-core model. This is joint work with Mihyun Kang, Michail Sarantis, and Ronen Wdowinski.

On Kotzig's conjecture in random graphs (Amadeo Sgueglia, University of Passau)

In 1963, Anton Kotzig famously conjectured that K_n, the complete graph of order n, where n is even, can be decomposed into $n-1$ perfect matchings such that every pair of these matchings forms a Hamilton cycle. The problem is still wide open and here we consider a variant of it for the binomial random graph $G(n, p)$. In fact, our main result is a very precise answer for the following counting problem: given any k edge-disjoint perfect matchings $M_1, \ldots, M_k$ of K_n, how many perfect matchings M^* in K_n have the property that $M^* \cup M_i$ forms a Hamilton cycle for each $i \in [k]$?

Joint work with Stefan Glock.

The maximum diameter of d-dimensional simplicial complexes (Silas Rathke, FU Berlin)

We consider the problem of Santos about finding the largest possible diameter of a strongly connected d-dimensional simplicial complex on n vertices. For $d = 2$, we determine the answer precisely for every n by an explicit construction. For larger d, we find the optimum value for every n that is large enough. The underlying theorem can be seen as a generalization of finding a tight Euler trail in a d-uniform hypergraph. Joint work with Stefan Glock, Olaf Parczyk, and Tibor Szabó.

The Ramsey Multiplicity Problem (Jae-baek Lee, University of Victoria)

A graph H is said to be *common* if the number of monochromatic copies of H is asymptotically minimized by a random colouring. It is well known that the disjoint union of two common graphs may be uncommon; e.g., K_2 and K_3 are common, but their disjoint union is not. We investigate the commonality of disjoint unions of multiple copies of K_3 and K_2. As a consequence of our results, we obtain the first example of a pair of uncommon graphs whose disjoint union is common. Our approach is to reduce the problem of showing that certain disconnected graphs are common to a constrained optimization problem in which the constraints are derived from supersaturation bounds related to Razborov's Triangle Density Theorem. We also improve the bounds on the Ramsey multiplicity constant of a triangle with a pendant edge and the disjoint union of K_3 and K_2. Fox and Wigderson recently identified a large family of graphs whose Ramsey multiplicity constants are attained by sequences of "Turán colourings;" i.e. colourings in which one of the colour classes forms the edge set of a balanced complete multipartite graph. Each graph in their family comes from taking a connected non-3-colourable graph with a critical edge and adding many pendant edges. We focus on finding smaller graphs whose Ramsey multiplicity constants are achieved by Turán colourings. While Fox and Wigderson provide many examples, their smallest constructions involve graphs with at least 10^{66} vertices. In contrast, we identify a graph on only 10 vertices whose Ramsey multiplicity constant is achieved by Turán colourings. To prove this, we apply the method developed eariler and used a powerful technique known as the flag algebra method, assisted by semi-definite programming.

Double-jump phase transition for the reverse Littlewood–Offord problem (Victor Souza, Cornell University)

Erd″s conjectured in 1945 that for any unit vectors $v_1, \ldots, v_n$ in $\mathbb{R}^2$ and signs $\varepsilon_1, \ldots, \varepsilon_n$ taken independently and uniformly in $\{-1, 1\}$, the random Rademacher sum $\sigma = \varepsilon_1 v_1 + \ldots + \varepsilon_n v_n$ satisfies $\|\sigma\|_2 \leq 1$ with probability $\Omega(1/n)$. While this conjecture is false for even n, Beck has proved that $\|\sigma\|_2 \leq \sqrt{2}$ always holds with probability $\Omega(1/n)$. Recently, He, Juškevičius, Narayanan, and Spiro conjectured that the Erdős' conjecture holds when n is odd. We disprove this conjecture by exhibiting vectors $v_1, \ldots, v_n$ for which $\|\sigma\|_2 \leq 1$ occurs with probability $O(1/n^{3/2})$. On the other hand, an approximated version of their conjecture holds: we show that we always have $\|\sigma\|_2 \leq 1+\delta$ with probability $\Omega_\delta(1/n)$, for all $\delta > 0$. This shows that when n is odd, the minimum probability that $\|\sigma\|_2 \leq r$ exhibits a double-jump phase transition at $r = 1$, as we can also show that $\|\sigma\|_2 \leq 1$ occurs with probability at least $\Omega((1/2+\mu)^n)$ for some $\mu > 0$. Joint work with L. Hollom and J. Portier.

Geometric hypergraphs (Gábor Damásdi, Renyi Institute)

In this talk I will give a brief introduction to the world of geometric hypergraphs. These are the hypergraps that we can get the following way: take a set of points in the plane and a set of planar objects (such as squares, disk, rectangles ect). The base set of the hypergraph is the pointset and each object defines an edge by incidence. What can we say about the combinatorial properties of these hypergraphs, such as their chromatic number? I will highlight some central results and show some nice open problems.