2025 Developments in Combinatorics Workshop

Grand Hyatt Jeju

Jeju Island, South Korea

October 31-November 3, 2025

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development-in-combinatorics-5th-workshop-2025/

2025 Developments in Combinatorics Workshop

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Organizing Committee

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Sponsored by

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Schedule

- **October 31, Friday**

  Registration day

  - 15:00-18:00 Free discussion

  Banquet

  - 19:00-21:00

- **November 1, Saturday**

  Session 1A Talks

  - 9:00-9:10 Talk 1A-1
  - 9:10-9:20 Talk 1A-2
  - 9:20-9:30 Talk 1A-3
  - 9:30-9:40 Talk 1A-4
  - 9:40-9:50 Talk 1A-5
  - 9:50-10:00 Talk 1A-6

  Coffee Break

  Session 1B Talks

  - 10:30-10:40 Talk 1B-1
  - 10:40-10:50 Talk 1B-2
  - 10:50-11:00 Talk 1B-3
  - 11:00-11:10 Talk 1B-4
  - 11:10-11:20 Talk 1B-5
  - 11:20-11:30 Talk 1B-6

  Session 1C Talks

  - 14:00-14:10 Talk 1C-1
  - 14:10-14:20 Talk 1C-2
  - 14:20-14:30 Talk 1C-3
  - 14:30-14:40 Talk 1C-4
  - 14:40-14:50 Talk 1C-5
  - 14:50-15:00 Talk 1C-6

Session 1D Discussions

- 15:00-17:00 Free discussion

Banquet

- 18:00-20:00

November 2, Sunday

Session 2A Talks

- 9:00-9:10 Talk 2A-1
- 9:10-9:20 Talk 2A-2
- 9:20-9:30 Talk 2A-3
- 9:30-9:40 Talk 2A-4
- 9:40-9:50 Talk 2A-5
- 9:50-10:00 Talk 2A-6

Coffee Break

Session 2B Talks

- 10:30-10:40 Talk 2B-1
- 10:40-10:50 Talk 2B-2
- 10:50-11:00 Talk 2B-3
- 11:00-11:10 Talk 2B-4
- 11:10-11:20 Talk 2B-5
- 11:20-11:30 Talk 2B-6

Session 2C Talks

- 14:00-14:10 Talk 2C-1
- 14:10-14:20 Talk 2C-2
- 14:20-14:30 Talk 2C-3
- 14:30-14:40 Talk 2C-4
- 14:40-14:50 Talk 2C-5

Session 2D Discussions

- 15:00-17:00 Free discussion

November 3, Monday

- 9:00-12:00 Free discussion

Distance-regular graphs without 5-claws

Proposed by Sejeong Bang, Yeungnam University

A t-claw is a complete bipartite graph $K_{1,t}$ with parts of size 1 and t. We say that a graph is t-claw-free (or without t-claws), if it does not contain a t-claw as an induced subgraph. Clearly, a connected graph without 2-claws is a complete graph. Blokhuis and Brouwer [2] and (independently) Kabanov and Makhnev [3] determined all distance-regular graphs without 3-claws. In [1], we determined the 4-claw-free distance-regular graphs with diameter at least 3 and $c_2 \geq 2$.

Problem 1. *For an integer $t \geq 5$, classify distance-regular graphs without t-claws.*

References

[1] S. Bang, A. Gavrilyuk, J. Koolen, Distance-regular graphs without 4-claws, *European J. Combin.* **80 (2019)** 120–142.

[2] A. Blokhuis, A.E. Brouwer, Determination of the distance-regular graphs without 3-claws, *Discrete Math.* **163 (1–3)** (1997) 225–227.

[3] V.V. Kabanov, A.A. Makhnev, On separated graphs with certain regularity conditions, *Mat. Sb.* **187 (10)** (1996) 73–86 (in Russian).

Talagrand-type correlation inequalities and Chvátal's conjecture

Proposed by Fan Chang, Nankai University & ECOPRO

Correlation inequalities between increasing functions play an important role in numerous areas, including probability, combinatorics, mathematical physics, etc. In this subsection we consider increasing functions defined on the discrete cube $\{0,1\}^n$ endowed with the uniform measure μ, and especially Boolean functions that can be treated as characteristic functions of subsets of the discrete cube.

Definition 1. *A function $f : \{0,1\}^n \to \mathbb{R}$ is* increasing *if for all $x, y \in \{0,1\}^n$,*

$$x_i \le y_i,\ \forall i \in [n] \Rightarrow f(x) \le f(y).$$

A family $\mathcal{F} \subset \{0,1\}^n$ is called increasing if its characteristic function f is increasing.

One of the best-known correlation inequalities is Harris–Kleitman inequality which asserts that any two increasing families $\mathcal{F}, \mathcal{G} \subset \{0,1\}^n$ are nonnegatively correlated, i.e., satisfy

$$\mathbb{E}_\mu[fg] - \mathbb{E}_\mu[f]\mathbb{E}_\mu[g] \ge 0,$$

Talagrand [4] proposed to measure the simultaneous dependence of f and g on coordinates via the *cross-total-influence of f, g*

$$\mathrm{I}[f,g] := \sum_{i=1}^{n} \mathrm{Inf}_i[f]\mathrm{Inf}_i[g], \quad \text{where } \mathrm{Inf}_i[f] \ := \ \mathbb{P}_\mu\left[f(x) \ne f(x \oplus e_i)\right].$$

and asked whether one can lower bound the covariance by $\mathrm{I}[f,g]$. Without additional structure, however, the dreaming inequality fails for general increasing pairs. Talagrand presented the following lower bound and gave a tight example that the log term cannot be removed in general:

$$\mathbb{E}[fg] - \mathbb{E}[f]\mathbb{E}[g] \ge C \cdot \frac{\mathrm{I}[f,g]}{\log\left(e/\mathrm{I}[f,g]\right)}.$$

Problem 2 (Kalai–Keller–Mossel [2])**.** *For any two increasing Boolean functions $f, g : \{0,1\}^n \to \{0,1\}$, find additional conditions to be such that the following inequality holds:*

$$\mathbb{E}[fg] - \mathbb{E}[f]\mathbb{E}[g] \ge c \cdot \sum_{i=1}^{n} \mathrm{Inf}_i[f]\mathrm{Inf}_i[g], \tag{1}$$

where $c > 0$ is a universal constant.

The first positive evidence came in an *average-case* form: Keller [3] proved that the analogue of (1) holds with $c = 1$ after averaging over all pairs in a family $\mathcal{T}$ of increasing Boolean functions. A second, conceptually different, reduction isolates the *antipodal* condition $g(x) = 1 - g(1-x)$, which forces $\mathbb{E}[g] = \frac{1}{2}$ and links the problem to extremal set theory: Friedgut, Kahn, Kalai, and Keller [1] showed that the following conjecture is equivalent to the celebrated Chvátal conjecture:

Conjecture 3 (Friedgut–Kahn–Kalai–Keller [1])**.** *If $f, g : \{0,1\}^n \to \{0,1\}$ are increasing and g is antipodal, then*

$$\mathbb{E}[fg] - \mathbb{E}[f]\mathbb{E}[g] \ge \tfrac{1}{4} \cdot \min_i \mathrm{Inf}_i[f]. \tag{2}$$

Moreover, a routine application of Harper's edge-isoperimetric inequality ($\mathrm{I}[g] \ge 2\alpha \log_2(1/\alpha)$ for $\alpha = \mathbb{E}[g] \le \frac{1}{2}$) shows that a cross-total-influence lower bound of the form $\mathbb{E}[fg] - \mathbb{E}[f]\mathbb{E}[g] \ge \frac{1}{4}\mathrm{I}[f,g]$ already implies a Chvátal-type estimate $\mathbb{E}[fg] - \mathbb{E}[f]\mathbb{E}[g] \ge \frac{\alpha}{2}\log_2(\frac{1}{\alpha}) \cdot \min_i \mathrm{Inf}_i[f]$ (and hence $\frac{1}{4}\min_i \mathrm{Inf}_i[f]$ when g is antipodal).

References

[1] Ehud Friedgut, Jeff Kahn, Gil Kalai, and Nathan Keller. Chvátal's conjecture and correlation inequalities. *J. Combin. Theory Ser. A*, 156:22–43, 2018.

[2] Gil Kalai, Nathan Keller, and Elchanan Mossel. On the correlation of increasing families. *J. Combin. Theory Ser. A*, 144:250–276, 2016.

[3] Nathan Keller. Lower bound on the correlation between monotone families in the average case. *Adv. in Appl. Math.*, 43(1):31–45, 2009.

[4] Michel Talagrand. How much are increasing sets positively correlated? *Combinatorica*, 16(2): 243–258, 1996.

Rational exponents for generalized Turán numbers

Proposed by Laihao Ding, Central China Normal University

A rational number r is a **realizable exponent** for a graph H if there exists a finite family of graphs $\mathcal{F}$ such that $\mathrm{ex}(n, H, \mathcal{F}) = \Theta(n^r)$, where $\mathrm{ex}(n, H, \mathcal{F})$ denotes the maximum number of copies of H that an n-vertex $\mathcal{F}$-free graph can have.

The following results were proved.

Theorem 1 ([2])**.** *Every rational* $r \in [1, t]$ *is realizable for* K_t*.*

Let S_t be the star with t edges.

Theorem 2 ([2])**.** *The rationals realizable for* S_t *are precisely* $0, 1$ *and those in* $[t, t+1]$*.*

Theorem 3 ([1])**.** *Let* $t \geq 2$ *and let* H *be a non-empty graph on* t *vertices. Then every rational* $r \in [t-1, t]$ *is realizable for* H*.*

Theorem 4 ([3])**.** *If* H *is a graph with maximum degree* $\Delta \geq 1$*, then every rational number in the interval*

$$\left[v(H) - \frac{e(H)}{2\Delta^2}, v(H)\right]$$

is realizable for H*.*

English and Spiro [3] asked the following question.

Problem 5. *For every graph* H*, is every rational in* $[\alpha(H), v(H)]$ *realizable for* H*?*

References

[1] B. van der Beek and A. Bishnoi. Rational exponents for generalized Turán numbers. *arXiv: 2510.19621*, 2025.

[2] Sean English, Anastasia Halfpap, and Robert A. Krueger. Rational exponents for cliques. *SIAM J. Discrete Math.*, 39(2):1246–1268, 2025.

[3] Sean English and Sam Spiro. Rational exponents for general graphs. *arXiv: 2506.19061*, 2025.

Covering a Graph by Perfect Matchings

Proposed by Genghua Fan, Fuzhou University

An edge e of a graph G is *covered* by a subgraph H of G if e is contained in H. A graph is *matching covered* if each edge is covered by a perfect matching. For a matching covered graph G, how many perfect matchings are needed to cover all the edges of G? Denote by $mc(G)$ the minimum number of perfect matchings needed to cover all the edges of G.

Problem 1. *Let G be a matching covered k-regular graph. Is there a function $f(k)$ such that $mc(G) \leq f(k)$?*

For $k = 3$, the Berge-Fulkerson Conjecture claims that $f(k) = 2k - 1 = 5$.

The threshold of Planar graphs

Proposed by Jie Han, Beijing Institute of Technology

By Euler's formula, every planar graph G satisfies that $e_G \leq 3v_G - 6$. This key property, together with a second moment argument of Riordan, implies that the threshold of planar graphs (as spanning subgraphs) is $n^{-1/3}$. That is, if H is an n-vertex planar graph with bounded maximum degree and $p \gg n^{-1/3}$, then $G(n,p)$ whp contains a spanning copy of H. There are two natural extension/strengthening of this result.

- (Perturbation threshold) If G is an n vertex graph with $\delta(G) \geq \varepsilon n$ and a copy of $G(n,p)$ is placed on $V(G)$ with $p \geq n^{-1/3-o(1)}$, does $G \cup G(n,p)$ whp contain a spanning copy of H?
- (Universality for planar graphs) Let $P(n,\Delta)$ be the family of all n-vertex planar graphs of maximum degree Δ. Suppose $p \geq n^{-1/3+\varepsilon}$, does $G(n,p)$ whp contain all members of $P(n,\Delta)$ simultaneously?

The current record for Universality, is $n^{-1/10}$ times a polylog factor, by the degeneracy bound obtained by Ferber–Nenadov, and that planar graphs are 5-degenerate.

Problem

Proposed by Eng Keat Hng, IBS ECOPRO

The kth power G^k of a graph G is obtained from G by adding edges between all pairs of vertices at distance at most k in G. Espuny Díaz, Lichev and Wesolek [2] posed the following conjecture. This is motivated by the study of local resilience in toroidal random geometric graphs with respect to Hamiltonicity.

Conjecture 1. *For all integers $n \geq 3$ and $1 \leq k \leq \frac{n}{2}$, every graph $H \subseteq C_n^k$ with $\delta(H) \geq k+1$ is Hamiltonian.*

The case $k = 1$ is trivial, and the case $k \geq \frac{n}{2} - 1$ follows from Dirac's theorem. In [2] the first non-trivial case of Conjecture 1 is proved.

Proposition 2 ([2])**.** *For every integer $n \geq 4$, every graph $H \subseteq C_n^2$ with $\delta(H) \geq 3$ is Hamiltonian.*

In [1] a variant of Conjecture 1 was considered, with P_n^k instead of C_n^k, and the following theorem was proved.

Theorem 3 ([1])**.** *For all integers $n \geq 4$ and $k \geq 2$, every graph $H \subseteq P_n^k$ with $\delta^*(H) \geq k+2$ is Hamiltonian.*

References

[1] A. Espuny Díaz, P. Gupta, D. Mergoni Cecchelli, O. Parczyk, and A. Sgueglia, Dirac's theorem for graphs of bounded bandwidth, arXiv:2407.05889, 2024.

[2] A. Espuny Díaz, L. Lichev, and A. Wesolek, On the local resilience of random geometric graphs with respect to connectivity and long cycles, arXiv:2406.09921, 2024.

Problem

Proposed by Jianfeng Hou, Fuzhou University

Given a graph G, a *cut* in G is a bipartition (U, V) of the vertex set together with all the edges having exactly one endpoint in both parts. The *size* of the cut is the number of its edges. The $MaxCut$ of G is the maximum size of a cut, denoted by $\mathrm{mc}(G)$. For a graph G with m edges, a random bipartition shows that $\mathrm{mc}(G) \geq m/2$. Answering a question of Erdős, Edwards improved this bound to

$$m/2 + (\sqrt{8m+1} - 1)/8,$$

which is tight for complete graphs of odd order. It is natural to study the MaxCut of H-free graphs for a fixed graph H. In 2003, Alon, Bollobás, Krivelevich and Sudakov suggested the following general conjecture.

Conjecture 1. *For any fixed graph H, there is a constant $\epsilon(H) > 0$ such that*

$$\mathrm{sp}(m, H) = m/2 + \Omega(m^{3/4+\epsilon(H)}).$$

Combinatorial interpretations of congruences for the spt-function

Proposed by Qing Ji, Tianjin University

The spt-function $spt(n)$ was introduced by Andrews [1] as the weighted counting of partitions of n with respect to the number of occurrences of the smallest part. For example, $spt(4) = 10$ as shown in the following table:

λ	(4)	$(3,1)$	$(2,2)$	$(2,1,1)$	$(1,1,1,1)$
$n_s(\lambda)$	1	1	2	2	4

Andrews [1] proved that $spt(n)$ satisfies the following congruences mod 5, 7 and 13, reminiscent to Ramanujan's congruences for $p(n)$, the number of partitions of n:

$$\begin{aligned} spt(5n+4) &\equiv 0 \pmod 5, \\ spt(7n+5) &\equiv 0 \pmod 7 \\ spt(13n+6) &\equiv 0 \pmod{13}. \end{aligned}$$

A classical combinatorial problem is to find combinatorial interpretations of these three congruences, that is,

Problem 1. *Find a statistic defined on ordinary partitions so that the set of partitions counted by $spt(5n+4)$ (resp. $spt(7n+5)$, $spt(13n+6)$) can be divided into five (resp. seven, thirteen) equinumerous classes.*

Andrews, Garvan and Liang [3] defined the spt-crank of a vector partition which leads to combinatorial interpretations of the congruences of $spt(n)$ mod 5 and 7 and asked for a definition of the spt-crank for ordinary partitions. Andrews, Dyson and Rhoades [2] speculated that "It is an *interesting and apparently challenging problem* to interpret the spt-crank in terms of ordinary integer partitions". Chen, Ji and Zang [4] provided a solution to the problem of Andrews, Dyson and Rhoades by introducing the spt-crank of a doubly marked partition and constructing a bijection between marked partitions and doubly marked partitions.

The story, however, is far from complete. A subtle but significant gap lingers: *It is still interesting to find an spt-crank that can be directly defined on the ordinary partitions. Moreover, there is currently no information on the combinatorial interpretation of the congruences of $spt(n)$ mod 13.*

References

[1] G.E. Andrews, The number of smallest parts in the partitions of n, J. Reine Angew. Math. 624 (2008) 133–142.

[2] G.E. Andrews, F.J. Dyson and R.C. Rhoades, On the distribution of the spt-crank, MDPI-Mathematics 1 (3) (2013) 76–88.

[3] G.E. Andrews, F.G. Garvan, J.L. Liang, Combinatorial interpretations of congruences for the spt-function. Ramanujan J. 29, (2012) 321–338.

[4] W. Y.C. Chen, Kathy Q. Ji, and Wenston J.T. Zang, The spt-crank for ordinary partitions, J. Reine Angew. Math. 711 (2016), 231–249.

K_r-factor in graphs with low ℓ-independence number

Proposed by Suyun Jiang, Jianghan University

For an integer $\ell \geq 2$ and a graph G, the *ℓ-independence number* of G, denoted by $\alpha_\ell(G)$, is the maximum size of a K_ℓ-free subset of vertices (the independence number of G corresponds to $\alpha_2(G)$). Given integers n, r and a function $f(n)$, we use $\mathrm{RT}_\ell(n, K_r, f(n))$ to denote the maximum number of edges of an n-vertex K_r-free graph G with $\alpha_\ell(G) \leq f(n)$. In particular, the *Ramsey–Turán density* of K_r is defined as

$$\varrho_\ell(r) := \lim_{\alpha \to 0} \lim_{n \to \infty} \frac{\mathrm{RT}_\ell(n, K_r, \alpha n)}{\binom{n}{2}}.$$

In 2016, Balogh, Molla and Sharifzadeh [1] proposed the following Ramsey–Turán type problem.

Problem 1 ([1])**.** *Let $r \geq 3$ be an integer and G be an n-vertex graph with $\alpha(G) = o(n)$. What is the minimum degree condition on G that guarantees a K_r-factor?*

Known results:

(1) $r = 3$, $\delta(G) \geq \frac{n}{2} + \varepsilon n$ for any $\varepsilon > 0$ (Balogh, Molla and Sharifzadeh [1])

(2) $r \geq 4$, $\delta(G) \geq (1 - \frac{2}{r})n + o(n)$ (Knierim and Su [4])

In 2020, Nenadov and Pehova [5] proposed the following problem.

Problem 2 ([5])**.** *For all $r, \ell \in \mathbb{N}$ with $r \geq \ell \geq 2$, let G be an n-vertex graph with $n \in \mathbb{N}$ and $\alpha_\ell(G) = o(n)$. What is the best possible minimum degree condition on G that guarantees a K_r-factor?*

Known results:

(1) $r = \ell + 1$, $\delta(G) \geq n/2 + o(n)$ (Nenadov, Pehova [5])

(2) $r > \ell \geq \frac{3}{4}r$, $\delta(G) \geq \left(\frac{1}{2 - \varrho_\ell(r-1)} + \mu\right) n$ (Chang, Han, Kim, Wang and Yang [2])

References

[1] J. Balogh, T. Molla, M. Sharifzadeh, Triangle factors of graphs without large independent sets and of weighted graphs, *Random Struct. Algorithms* 49 (4) (2016) 669–693.

[2] F. Chang, J. Han, J. Kim, G. Wang, D. Yang, Embedding clique-factors in graphs with low-ℓ-independence number, *J. Comb. Theory Ser. B* 161 (2023) 301–330.

[3] M. Chen, J. Han, G. Wang, D. Yang, H-factors in graphs with small independence number, *J. Comb. Theory Ser. B* 169, (2024) 373-405.

[4] C. Knierim, P. Su, K_r-factors in graphs with low independence number, *J. Comb. Theory Ser. B* 148 (2021) 60–83.

[5] N. Nenadov, T. Pehova, On a Ramsey–Turán variant of the Hajnal–Szemerédi theorem, *SIAM J. Discrete Math.* 34 (2) (2020) 1001–1010.

A recursive formula for the polymatroid Tutte polynomial

Proposed by Xian'an Jin, Xiamen University

The main use of deletion-contraction, for graphs and, to a lesser degree, matroids, is to compute the Tutte polynomial recursively. Our deletion-contraction formula we have obtained in [1] is a sum over parallel slices of a polymatroid. It operates within the category of polymatroids and complements the activity-based definition and the state sum given in [2, Theorem 10.6] (whose underlying idea is a version of the Crapo interval subdivision) with a third, recursive approach.

However, even if a polymatroid is hypergraphical, we do not see a natural reason for all of its slices, apart from the two extremal ones, to be themselves hypergraphical. Hence, we pose the following question.

Problem 1. *Are all slices of a hypergraphical polymatroid also hypergraphical polymatroids?*

A positive answer would be desirable in order to have a recursive method of computation that is fully within the realm of hypergraphs. In case the answer is negative, which appears likely, we relax our question as follows.

Problem 2. *Can we express $\mathscr{T}_{P_{\mathcal{H}}}(x,y)$, for an arbitrary hypergraph $\mathcal{H}$, in terms of polymatroid Tutte polynomials of some (in a suitable sense) smaller hypergraphs?*

References

[1] X. Guan, X. Jin and T. Kálmán, *A deletion-contraction formula and monotonicity properties for the polymatroid Tutte polynomial*, Int. Math. Res. Not. IMRN 2025, no. 19, Art. rnaf302, 16 pp.

[2] O. Bernardi, T. Kálmán and A. Postnikov, *Universal Tutte polynomial*, Adv. Math. 402 (2022), 108355.

Relation of $\mathbf{SPEX}_p(n, \mathcal{F})$ and $\mathbf{EX}(n, \mathcal{F})$

Proposed by Liying Kang, Shanghai University

For any real number $p \geq 1$, the p-spectral radius of hypergraphs was introduced by Keevash, Lenz and Mubayi and subsequently studied by Nikiforov. Given an r-graph H of order n, the *p-spectral radius* of H is defined as:

$$\lambda^{(p)}(H) := \max_{\|\boldsymbol{x}\|_p=1} r! \sum_{e \in E(H)} \prod_{v \in e} x_v,$$

where $\|\boldsymbol{x}\|_p := (|x_1|^p + \cdots + |x_n|^p)^{1/p}$.

Problem 1 (Keevash-Lenz-Mubayi 2014)**.** *Let $p \geq r \geq 3$, and $\mathcal{F}$ be a family of r-graphs. What is the maximum p-spectral radius of an r-graph H on n vertices without member of $\mathcal{F}$ as a subgraph?*

We denote by $\mathrm{SPEX}_p(n, \mathcal{F})$ the class of r-graphs that attain the maximum p-spectral radius among all $\mathcal{F}$-free n-vertex r-graphs.

Problem 2. *Let $p > r - 1$ and $\mathcal{F}$ be a family of r-graphs. Characterize all $\mathcal{F}$ such that*

$$SPEX_p(n, \mathcal{F}) \subseteq EX(n, \mathcal{F})$$

for sufficiently large n.

Problem 3 (Keevash-Lenz-Mubayi 2014)**.** *Let $p > r - 1$ and $\mathcal{F}$ be a family of r-graphs. Characterize all $\mathcal{F}$ such that*

$$SPEX_p(n, \mathcal{F}) \cap EX(n, \mathcal{F}) = \emptyset$$

for sufficiently large n.

References

[1] P. Keevash, J. Lenz, D. Mubayi, Spectral extremal problems for hypergraphs, *SIAM J. Discrete Math.* **28** (4) (2014) 1838–1854.

Problem

Proposed by Seog-Jin Kim, Konkuk University

Dvořák et. al [2] and Feder et. al [3] conjectured the following.

Conjecture 1. *Is it true that* $\chi(G^2) \leq 6$ *if* G *is a cubic bipartite planar graph?*

One may see recent study of coloring of the square of subcubic planar graphs in [1]. Feder et. al [3] showed that Conjecture 1 is true for very special case of cubic bipartite planar graph. As a natural direction, we can ask the following question.

Problem 2. *Is it true that* $\chi_\ell(G^2) \leq 6$ *if* G *is a cubic bipartite planar graph?*

It is even not known whether the square of a cubic bipartite planar graph is 7-choosable or not.

References

[1] D. W. Cranston, Coloring, List Coloring, and Painting Squares of Graphs (and Other Related Problems), Electron. J. Combin. 30(2) (2023), #DS25.

[2] Z. Dvořák, R. Škrekovski, and M. Tancer, List-coloring squares of sparse subcubic graphs, SIAM J. Discrete Math. 22 (2008), no. 1, 139–159

[3] T. Feder, P. Hell, and C. Subi, Distance-two colourings of Barnette graphs, European J. Combin. 91 (2021), Paper No. 103210.

Problem

Proposed by Young Soo Kwon, Yeungnam University

Let G be a group and S a subset of G with $1 \notin S$. The Cayley digraph $\mathrm{Cay}(G, S)$ of G with S is defined to have vertex set G and arc set $\{(g, sg) \mid g \in G, s \in S\}$. If S is inverse-closed, that is, $S = S^{-1}$, then $\mathrm{Cay}(G, S)$ is a graph. Two Cayley digraphs $\mathrm{Cay}(G, S)$ and $\mathrm{Cay}(G, T)$ are called *Cayley isomorphic* if there is an $\alpha \in \mathrm{Aut}(G)$ such that $S^\alpha = T$. Cayley digraphs are isomorphic if they are Cayley isomorphic, but the converse is not true. A subset S of G with $1 \notin S$ is said to be a *CI-subset* if for every $\mathrm{Cay}(G, T)$ isomorphic to $\mathrm{Cay}(G, S)$, they are Cayley isomorphic, and in this case, $\mathrm{Cay}(G, S)$ is called a *CI-digraph*, or a *CI-graph* when $S = S^{-1}$. For a positive integer m, if all Cayley digraphs of G with out-valency m are CI-digraphs, then G is said to have the *m-DCI property*, and if all Cayley graphs of G with valency m are CI-graphs, then G is said to have the *m-CI property*. Clearly, m-DCI property implies m-CI property. A group G is called an *m-DCI-group* or *m-CI-group* if G has the k-DCI property or k-CI property for all positive integers $k \leq m$, respectively. Furthermore, G is called a *DCI-group* or *CI-group*, if G has the m-DCI property or m-CI property for all positive integers m, respectively.

In 1967, Ádám [1] conjectured that every finite cyclic group is a CI-group. Even this conjecture was disproved by Elspas and Turner [2], it stimulated the study of DCI-groups and CI-groups. By Muzychuk [4, 5], it was shown that a cyclic group C_n of order n is a DCI-group if and only if n is equal to mk such that $m \in \{1, 2\}$ and k is a square-free integer, and C_n is a CI-group if and only if either C_n is a DCI-group or $n \in \{8, 9, 18\}$. Despite the wealth of partial discoveries surrounding dihedral CI-groups and DCI-groups, the classification of dihedral CI-groups and DCI-groups remains a formidable challenge.

For a positive integer n, write $\mathrm{D}_{2n} = \langle a, b \mid a^n = b^2 = 1,\ a^b = a^{-1}\rangle$, the dihedral group of order $2n$. Denote by $\mathbb{Z}_n$ the additive group of integers modulo n. Since automorphism group of a digraph is the same with that of its complement digraph, for the m-DCI property of a dihedral group D_{2n}, it suffices to consider m such that $1 \leq m \leq n-1$. The following is some recent result related to the m-DCI property of a dihedral group D_{2n} by Jin-Hua Xie, Yan-Quan Feng and me [3].

Theorem 1. *Let G be a dihedral group of order $2n$ with an integer $n \geq 3$. Then the following statements hold:*

(1) *Let G have the m-DCI property for some $1 \leq m \leq n-1$. Then n is odd, and if further $p+1 \leq m \leq n-1$ for a prime divisor p of n, then $G_p \cong \mathbb{Z}_p$;*

(2) *Let n be a power of a prime q and let $1 \leq m \leq n-1$. Then G has the m-DCI property if and only if q is odd, and either $n = q$ or $1 \leq m \leq q$.*

Based on the above Theorem, we have the following corollary.

Corollary 2. *Let G be a dihedral group of order $2n$ with an integer $n \geq 2$. If G is a DCI-group, then $n = 2$ or n is odd and square-free.*

We conjecture that the converse of the above Corollary is true.

Conjecture 3. *For a dihedral group G of order $2n$ with an integer $n \geq 2$, G is a DCI-group if and only if $n = 2$ or n is odd and square-free.*

For a Cayley digraph $\mathrm{Cay}(G, S)$, let an arc (g, sg) be called s-arc.

Remark 4. *Let n be a square-free odd integer. To prove the above conjecture is true, it suffices to show that for any Cayley digraph $\Gamma = \mathrm{Cay}(D_{2n}, S)$, $\mathrm{Aut}(\Gamma)$ preserves the set $\{x - arc \mid x \in S \cap \langle a \rangle\}$.*

References

[1] Ádám, A.: Research Problem 2-10. J. Combin. Theory 2, 393 (1967).

[2] Elspas, B., Turner, J.: Graphs with circulant adjacency matrices. J. Combin. Theory 9, 297–307 (1970).

[3] Xie, J.H, Feng, Y. Q and Kwon, Y. S: Dihedral groups with m-DCI-property. J. Algebraic Combin. 60, 73–86 (2024).

[4] Muzychuk, M.: Ádám's conjecture is true in the square-free case. J. Combin. Theory Ser. A 72, 118–134 (1995).

[5] Muzychuk, M.: On Ádám's conjecture for circulant graphs. Discrete Math. 176, 285–298 (1997).

Extremal numbers of hypergraphs with common links and non-Sidorenko hypergraphs

Proposed by Hyunwoo Lee, KAIST & IBS ECOPRO

For an r-partite r-graph F, define Sidorenko exponent $s(F)$ as

$$s(F) := \sup\{s \geq 0 : \exists r\text{-graph } H \text{ s.t. } t_F(H) = t_{K_r^{(r)}}(H)^s > 0\},$$

where $t_{H_1}(H_2)$ denotes the homomorphism density of H_1 in H_2. The celebrated Sidorenko's conjecture states that $s(F) = e(F)$ holds for every bipartite graph F. It is known that for all $r \geq 3$, the r-uniform version of Sidorenko's conjecture is false, and only a few hypergraphs are known to be Sidorenko.

For a $(r-1)$-partite $(r-1)$-graph H and an integer t, we denote by $H(t)$ the r-partite r-graph on t vertices on r-th part such that all of its links are the same H. Recently, I [2] discovered a new connection to the extremal number and the Sidorenko exponents as follows.

Theorem 1 (Lee [2])**.** *Let H be a $(r-1)$-partite $(r-1)$-graph and t be a positive integer. Then*

$$\mathrm{ex}(n, H(t)) \leq O_{H,t}\left(n^{r-\frac{1}{s(H)}}\right).$$

Also, I conjectured that the above upper bound would be tight for some large values of t.

Conjecture 2 (Lee [2])**.** *Let H be a $(r-1)$-partite $(r-1)$-graph. Then there is a constant t_0 that only depends on H such that for all $t \geq t_0$, we have*

$$\mathrm{ex}(n, H(t)) \geq \Omega_H\left(n^{r-\frac{1}{s(H)}}\right).$$

Very recently, Chen, Liu, Ye [1] confirmed the above conjecture for all Sidorenko hypergraphs.

Hence, we raise an open problem for this workshop on the smallest open case of the above conjecture. That is, what if we take H as a 3-uniform loose triangle?

Problem 3. *Let H be a 3-uniform loose triangle. Is there a positive integer $t_0 > 0$ such that the following holds for all $t \geq t_0$?:*

$$\mathrm{ex}(n, H(t)) \geq \Omega\left(n^{\frac{15}{4}}\right).$$

References

[1] Q. Chen, H. Liu, K. Ye, Extremal constructions for apex partite hypergraphs, arXiv:2510.07997, 2025.

[2] H. Lee, On Sidorenko exponents of hypergraphs, arXiv:2509.08680, 2025.

Correlation inequalities for colourings

Proposed by Joonkyung Lee, Yonsei University

Let $A, B \subseteq [q]$ and let H be a bipartite graph with bipartition $V_1 \cup V_2$. Denote by $c_{H,q}(A, B)$ the number of proper q-colourings of H such that every $v \in V_1$ uses colours in A and every $u \in V_2$ uses colours in B. We conjecture that the inequality

$$c_{H,q}(A, A)c_{H,q}(B, B) \leq c_{H,q}(A, B)c_{H,q}(B, A) \tag{1}$$

holds for any bipartite H and all $A, B \subseteq [q]$. We may assume either $A \subsetneq B$ or $B \subsetneq A$ (see [2]). In [2], the conjecture is verified for cycles, paths and complete bipartite graphs H. Furthermore, a suitable generalisation for complete multipartite graphs H has also been obtained. If the inequality holds, then the graph H can be used to obtain instances for Yufei Zhao's conjecture, stating that

$$\hom(G, K_q)^2 \leq \hom(G \times K_2, K_q)$$

holds for any graph G.

The inequality (1) is also reminiscent of a question of Jeff Kahn [1], whose particular case was again asked by Ron Peled and Yinon Spinka [3] as follows: let $x, y, z, w \in V_1$, where $V_1 \cup V_2$ is a bipartition of H. For a random q-colouring of H, does the inequality

$$\mathbb{P}[f(x) = f(y) = f(z) = f(w) = 1] \geq \mathbb{P}[f(x) = f(y) = 1]\mathbb{P}[f(z) = f(w) = 1]$$

hold?

References

[1] Jeff Kahn, Angelika Steger, and Benny Sudakov, *Oberwolfach Combinatorics workshop report*, 14 (2017), 5-81. doi: 10.4171/OWR/2017/1. http://publications.mfo.de/handle/mfo/3565

[2] Joonkyung Lee, Jaeseong Oh, and Jaehyeon Seo, *Counting homomorphisms in antiferromagnetic graphs via Lorentzian polynomials*. arxiv:2506.13659.

[3] Ron Peled and Yinon Spinka, *Three lectures on random proper colorings of* $\mathbb{Z}^d$. arXiv:2001.11566.

Some routing problems on windy weighted graphs

Proposed by Jianping Li, Yunnan University

In this talk, we propose some (arc) routing problems on windy weighted graphs. Concretely, given a windy connected graph $G = (V, E; w)$ of order n and size m, where $w : E \to \mathbb{R}^+$ is a windy cost function, *i.e.*, for each edge $v_i v_j \in E$, we denote $w(v_i, v_j)$ to be a cost to traverse this edge $v_i v_j$ from v_i to v_j and $w(v_j, v_i)$ to be a cost to traverse the same edge $v_i v_j$ (also $v_j v_i$) from v_j to v_i, for any two subsets V_0 and E_0 to satisfy $\phi \subseteq V_0 \subseteq V$ and $\phi \subseteq E_0 \subseteq E$, the routing problem on windy weighted graphs is asked to find a circuit C such that C visits each vertex in V_0 and traverses each edge in E_0 at least once, the objective is to minimize the cost of that circuit C.

Particularly, when $E_0 = E$, we refer this routing problem on windy weighted graphs as to the windy postman problem, and when $E_0 = \phi$ and $V_0 = V$, we refer this routing problem on windy weighted graphs as to the windy traveling salesman problem.

The constrained strong Steiner connectivity augmentation problem

Proposed by Junran Lichen, Beijing University of Chemical Technology

We propose the constrained strong Steiner connectivity augmentation (CSStCA) problem. Concretely, given a weighted digraph $D = (V, A; w)$, where $w : A \to \mathbb{R}^+$ is a weight function, K $(\subseteq V)$ is a set of k fixed vertices (called as terminals), and $T_k = (V_k, A_k)$ is a strongly connected Steiner subgraph (of D), *i.e.*, T_k contains a directed path from s to t, where $K \subseteq V_k$, s and t are any pair of two distinct terminals in K, we are asked to find an arc-set $A' \subseteq A \backslash A_k$ to satisfy the constraints that, given each arc $e \in A_k$, the reduced subgraph $D[A' \cup A_k \backslash \{e\}]$ is still a strongly connected Steiner subgraph (of D), the objective is to minimize the summation of weights of all arcs in A', where the summation is taken among all arc-sets to satisfy the constraints as mentioned-above.

Particularly, given a set $K = V$, we refer the CSStCA problem as to the constrained strong connectivity augmentation (CSCA) problem.

BES Problem

Proposed by Miao Liu, Shandong University & IBS ECOPRO

Given a family $\mathcal{F}$ of r-graphs, denote by $\mathrm{ex}(n,\mathcal{F})$ the *Turán number* of $\mathcal{F}$, i.e., the maximum number of edges in an n-vertex r-graph containing no element of $\mathcal{F}$ as a subgraph. We consider the family $\mathcal{F}^{(r)}(s,k)$ of all r-graphs with k edges and at most s vertices. In 1973, Brown, Erdős and Sós [1] introduced the function $f^{(r)}(n;s,k) := \mathrm{ex}(n;\mathcal{F}^{(r)}(s,k))$. They proved that

$$\Omega(n^{(rk-s)/(k-1)}) = f^{(r)}(n;s,k) = O(n^{\lceil (rk-s)/(k-1)\rceil}).$$

If the exponent is an integer t, then $s = k(r-t)+t$. If the exponent is not an integer, then even determining the order of magnitude of $f^{(r)}(n;s,k)$ is a major open problem. For instance, it includes the celebrated $(6,3)$-theorem of Ruzsa and Szemerédi, as well as the notoriously difficult $(7,4)$-problem. We focus on the case when the exponent is an integer. Then, it is natural to ask if the limit

$$\pi(r,t,k) := \lim_{n\to\infty} n^{-t} f^{(r)}(n;s,k)$$

exists. In several cases, subsequent works [2, 3, 5, 6, 7] have established the existence of the limit, although its exact value remains open. The natural remaining question is to determine the limiting values. Recently, Letzter and Sgueglia [4] showed that for sufficiently large r, when k is even, $\pi(r,t,k) = \frac{1}{t!}\binom{r}{t}^{-1}$ and when k is odd, $\pi(r,t,k) \le \frac{2}{t!}(2\binom{r}{t}-1)^{-1}$.

Problem 1. *For odd k and sufficiently large r,*

$$\lim_{n\to\infty} n^{-t} f^{(r)}(n,s,k) = \frac{2}{t!}\left(2\binom{r}{t}-1\right)^{-1}.$$

References

[1] WG Brown, P Erdős, and VT Sós. Some extremal problems on r-graphs, new directions in the theory of graphs (proc. third ann arbor conf., univ. michigan, ann arbor, mich, 1971), 1973.

[2] Michelle Delcourt and Luke Postle. The limit in the $(k+2,k)$-problem of Brown, Erdős and Sós exists for all $k \ge 2$. *Proceedings of the American Mathematical Society*, 152(05):1881–1891, 2024.

[3] Stefan Glock, Felix Joos, Jaehoon Kim, Marcus Kühn, Lyuben Lichev, and Oleg Pikhurko. On the (6, 4)-problem of Brown, Erdős, and Sós. *Proceedings of the American Mathematical Society, Series B*, 11(17):173–186, 2024.

[4] Shoham Letzter and Amedeo Sgueglia. On a problem of Brown, Erdős and Sós. *Proceedings of the American Mathematical Society*, 153(07):2729–2743, 2025.

[5] Vojtěch Rödl. On a packing and covering problem. *European Journal of Combinatorics*, 6(1):69–78, 1985.

[6] Chong Shangguan. Degenerate Turán densities of sparse hypergraphs II: a solution to the Brown-Erdős-Sós problem for every uniformity. *SIAM Journal on Discrete Mathematics*, 37(3):1920–1929, 2023.

[7] Chong Shangguan and Itzhak Tamo. Degenerate Turán densities of sparse hypergraphs. *Journal of Combinatorial Theory, Series A*, 173:105228, 2020.

Problem

Proposed by Ruifang Liu, Zhengzhou University

Conjecture 1 (Bollobás-Nikiforov, 2007, [1]). *Let G be an n-vertex K_{r+1}-free graph of size m with eigenvalues $\lambda_1 \geq \cdots \geq \lambda_n$. If G is not a complete graph, then*

$$\lambda_1^2 + \lambda_2^2 \leq 2(1 - \frac{1}{r})m.$$

Lin, Ning and Wu [2] made the first progress on this conjecture, i.e., they solved the case $r = 2$. Subsequently, Zhang [3] confirmed this conjecture for regular graphs. Recently, Zeng and Zhang [4] proved that the conjecture holds for both line graphs and graphs with at most $\frac{\sqrt{6}}{27}m^{\frac{3}{2}}$ triangles. For other cases, this conjecture is still open.

References

[1] B. Bollobás, V. Nikiforov, Cliques and the spectral radius, J. Combin. Theory Ser. B 97 (2007) 859-865.

[2] H.Q. Lin, B. Ning, B. Wu, Eigenvalues and triangles in graphs, Combin. Probab. Comput. 30 (2021) 258-270.

[3] S.T. Zhang, On the first two eigenvalues of regular graphs, Linear Algebra Appl. 686 (2024) 102-110.

[4] J.S. Zeng, X.-D. Zhang, A note on the Bollobás-Nikiforov conjecture, Linear Algebra Appl. 710 (2025) 230-242.

Let H be a color-critical graph with $\chi(H) = r+1 \geq 3$. Denote by $\mathrm{Ex}_{r+1}(n, H)$ (resp. $\mathrm{Spex}_{r+1}(n, H)$) the family of n-vertex H-free non-r-partite graphs with the maximum size (resp. spectral radius). Fang and Lin [8], Yu and Li [13] independently proposed the following conjecture.

Conjecture 2. *Let H be an arbitrary color-critical graph with $\chi(H) = r + 1 \geq 3$. For sufficiently large n,* $\mathrm{Spex}_{r+1}(n, H) \subseteq \mathrm{Ex}_{r+1}(n, H)$.

Conjecture 2 has been confirmed for C_3 ([1-3]), C_{2k+1} ([4,5]), books ([6,7]), $\theta(1, q, r)$ for even q ([8]), K_{r+1} ([9,10]) and generalized book graph ([11-13]).

References

[1] P. Erdős, Some theorems on graphs (in Hebrew), Riv. Lemat. 9 (1955) 13-17.

[2] L. Caccetta, R.-Z. Jia, Edge maximal non-bipartite graphs without odd cycles of prescribed lengths, Graphs Combin. 18 (2002) 75-92.

[3] H.Q. Lin, B. Ning, B. Wu, Eigenvalues and triangles in graphs, Combin. Probab. Comput. 30 (2021) 258-270.

[4] S.J. Ren, J. Wang, S.P. Wang, W.H. Yang, A stability result for C_{2k+1}-free graphs, SIAM J. Discrete Math. 38 (2024) 1733-1756.

[5] Z.Y. Zhang, Y.H. Zhao, A spectral condition for the existence of cycles with consecutive odd lengths in non-bipartite graphs, Discrete Math. 346 (2023) 113365.

[6] L. Miao, R.F. Liu, E.R. van Dam, Turán number of books in non-bipartite graphs, arXiv: 2508.12578.

[7] R.F. Liu, L. Miao, Spectral Turán problem of non-bipartite graphs: Forbidden books, European J. Combin., 2025, 126: 104136.

[8] L.F. Fang, H.Q. Lin, Non-bipartite graphs without theta subgraphs, 2025, arXiv: 2508.12855.

[9] A.E. Brouwer, Some lotto numbers from an extension of Turán's theorem, Report ZW152, Stichting Mathematisch Centrum, Amsterdam, 1981.

[10] Y.T. Li, Y.J. Peng, Refinement on spectral Turán's theorem, SIAM J. Discrete Math. 37 (2023) 2462-2485.

[11] B. Wang, W.W. Chen, P. Zhang, Non-r-partite graphs without complete split subgraphs, arXiv: 2508.12210.

[12] Y.T. Yu, S.C. Li, The exact Turán number of generalized book graph $B_{r,k}$ in non-r-partite graphs, arXiv: 2508.07533.

[13] Y.T. Yu, S.C. Li, Spectral Turán-type problem in non-r-partite graphs: Forbidden generalized book graph $B_{r,k}$, arXiv: 2508.12034.

The difference between the Randić index and matching number of graphs

Proposed by Suil O, The State University of New York, Korea

The *Randić index* of a graph G, written $R(G)$, is the sum of $\frac{1}{\sqrt{d(u)d(v)}}$ over all edges uv in $E(G)$. The concept was introduced by Milan Randić [5], which has a good correlation with several physicochemical properties of alkanes. Béla Bollobás and Erdős [3] generalized the Randić index by allowing the exponent to be any real number, creating general Randić index. The *matching number* of a graph G, written $\alpha'(G)$, is the maximum size of a matching in it.

In 2006, Aouchiche, Hansen, and Zheng conjectured an upper bound for the difference bewteen $R(G)$ and $\alpha'(G)$.

Conjecture 1. ([1]) *For an n-vertex connected graph G, we have*

$$R(G) - \alpha'(G) \leq \sqrt{\lfloor \frac{n+4}{7} \rfloor \lfloor \frac{6n+2}{7} \rfloor} - \lfloor \frac{n+4}{7} \rfloor;$$

equality holds only when G=$K_{p,q}$, where $p = \lfloor \frac{n+4}{7} \rfloor$ and $q = \lfloor \frac{6n+2}{7} \rfloor$.

Even if we drop the condition of "connectedness" in Conjecture 1, the bound may not be improved significantly.

Conjecture 2. *For an n-vertex graph G, we have*

$$R(G) - \alpha'(G) \leq \max\{\sqrt{\lfloor \frac{n+4}{7} \rfloor \lfloor \frac{6n+2}{7} \rfloor} - \lfloor \frac{n+4}{7} \rfloor, \left(\frac{\sqrt{6}-1}{7}\right) n\}.$$

If Conjecture 2 is true, then the upper bound is best possible since a disjoint union of $K_{1,6}$s or $K_{p,q}$ satisfies equality in the bound.

Possible Tools

Theorem 3. ([2]) *For a graph G, we have*

$$\alpha'(G) = \min_{S \subseteq V(G)} \frac{1}{2}(n - o(G-S) + |S|),$$

where for a graph H, $o(H)$ is the number of odd components in H.

Theorem 4. ([4]) *For an n-vertex graph G, we have*

$$R(G) = \frac{n}{2} - \sum_{uv \in E(G)} \frac{1}{2} \left(\frac{1}{\sqrt{(d(u)}} - \frac{1}{\sqrt{d(v)}} \right)^2.$$

References

[1] M. Aouchiche, P. Hansen, M. Zheng, Variable neighborhood search for extremal graphs. XVIII. Conjectures and results about the Randić index, *MATCH Commun. Math. Comput. Chem.* **56**(2006), 541–550.

[2] C. Berge, Sur le couplage maximum d'un graphe, *C. R. Acad. Sci. Paris* **247** (1958) 258–259.

[3] B. Bollobás, P. Erdős, A. Sarkar, Extremal graphs for weights, *Discrete Math.* **200**(1999), 5–19.

[4] G. Caporossi, I. Gutman, P. Hansen, L. Pavlović, Graphs with maximum connectivity index, *Comput. Biol. Chem.* **27**(2003), 85–90.

[5] M. Randić, On characterization of molecular branching, *J. Amer. Chem. Soc.* **97**(1975), 6609–6615.

Rank of design matrices

Proposed by Tuan Tran, University of Science and Technology of China

Very recently, Dadush, Eisenbrand, Pinchasi, Rothvoss, and Singer [1] showed that any simple rank d real-representable matroid which excludes a line of length l has at most $O(d^4 l)$ elements. This complements the tight bound of $(l-3)\binom{d}{2}+d$ for $l \geq 4$, due to Geelen, Nelson, and Walsh [3], which holds when the rank d is at least doubly exponential in l. These upper bounds are obtained by studying the rank of sparse design matrices.

Definition 1 (Design matrix)**.** *Let A be an $m \times n$ real matrix. Let $R_1, \ldots, R_m \in \mathbb{R}^n$ denote the rows of A, and let $C_1, \ldots, C_n \in \mathbb{R}^m$ denote the columns of A. We say that A is a (q,k,t)-design matrix if*

1. *For all $i \in [m]$, $|\mathrm{supp}(R_i)| \leq q$.*
2. *For all $j \in [n]$, $|\mathrm{supp}(C_j)| \geq k$.*
3. *For all $j_1 \neq j_2 \in [n]$, $|\mathrm{supp}(C_{j_1}) \cap \mathrm{supp}(C_{j_2})| \leq t$.*

Dvir, Saraf, and Wigderson [2] proved that any (q,k,t)-design matrix A satisfies $\mathrm{rank}(A) \geq n - \frac{ntq(q-1)}{k}$. Dadush et al. [1] improved the loss term by a factor of q via a careful accounting of column versus row sizes.

Problem 2. *Let A be a $(3,3k,6)$-design matrix. Is it true that*

$$\mathrm{rank}(A) \geq n - \frac{2n}{k} \ ?$$

An affirmative answer to the above problem would imply that any simple rank d real-representable matroid which excludes a line of length l has at most $O(d^2 l)$ elements, which is tight up to a constant multiplicative factor.

References

[1] Daniel Dadush, Friedrich Eisenbrand, Rom Pinchasi, Thomas Rothvoss, and Neta Singer. *Excluding a Line Minor via Design Matrices and Column Number Bounds for the Circuit Imbalance Measure.* `arXiv:2510.20301`, 2025.

[2] Z. Dvir, S. Saraf, and A. Wigderson. *Improved rank bounds for design matrices and a new proof of Kelly's theorem. Forum of Mathematics, Sigma*, vol. 2, p. e4. Cambridge University Press, 2014.

[3] J. Geelen, P. Nelson, and Z. Walsh. *Excluding a Line from Complex-Representable Matroids.* Vol. 303, American Mathematical Society, 2024.

Euler tours in hypergraphs

Proposed by Bin Wang, Beijing Institute of Technology

An *Euler tour* in a k-graph H is a sequence of (possibly repeating) vertices $v_1 \cdots v_m$ such that each k cyclically consecutive vertices forms an edge of H, and all edges of H appear uniquely in this way. Define the *Euler tour threshold* $\delta^{(k)}_{\text{Euler}}$ be the least $d > 0$ such that for every $\varepsilon > 0$, there exists n_0 such that any k-graph H on $n \geq n_0$ vertices with $\delta_{k-1}(H) \geq (d + \varepsilon)n$ such that every vertex degree of H is divisible by k admits an Euler tour. Chung, Diaconis, and Graham [1] conjectured that every large $K^{(k)}_n$ such that every vertex has degree divisible by k admits an Euler tour. Glock, Joos, Kühn, and Osthus [2] confirmed this conjecture and showed the existence of Euler tours in suitable hypergraphs by using results on cycle decompositions, which in particular show $\delta^{(k)}_{\text{Euler}} < 1$ for all k. For $k = 2$, it is easy to see that $\delta^{(2)}_{\text{Euler}} = 1/2$ (as $\delta(H) \geq |V(H)|/2$ is needed to ensure that the graph H is connected), and examples show that $\delta^{(k)}_{\text{Euler}} \geq 1/2$ holds for all $k \geq 3$ ([2]). The following conjecture for all $k \geq 3$ was posed.

Conjecture 1 (Glock, Kühn, and Osthus [3])**.** *For $k \geq 3$, $\delta^{(k)}_{\text{Euler}} \leq (k-1)/k$.*

It was first conjectured that $\delta^{(k)}_{\text{Euler}} = 1/2$ for all $k \geq 3$ in [2], but this was disproven by Piga, and Sanhueza-Matamala [4] by showing that $\delta^{(3)}_{\text{Euler}} = 2/3$.

References

[1] F. Chung, P. Diaconis, R. Graham, Universal cycles for combinatorial structures, Discrete Math., 110(1-3): 43-59, 1992.

[2] S. Glock, F. Joos, D. Kühn, D. Osthus, Euler tours in hypergraphs, Combinatorica, 40(5): 679-690, 2020.

[3] S. Glock, D. Kühn, D. Osthus, Extremal aspects of graph and hypergraph decomposition problem, In Surveys in combinatorics 2021, Volume 470 of London Math. Soc. Lecture Note Ser., 235-265, Cambridge Univ. Press, Cambridge, 2021.

[4] S. Piga, N. Sanhueza-Matamala, Cycle decompositions in 3-uniform hypergraphs, Combinatorica 43(1): 1-36, 2023.

ℓ_1-embeddability of planar graphs

Proposed by Guangfu Wang, Yantai University

Let $G = (V, E)$ be an undirected graph. Each assignment of non-negative weights to the edges of G naturally defines a metric space (V, μ), where for each pair of vertices $x, y \in V$, $\mu(x, y) = d_G(x, y)$ is the shortest-path distance between them. We say that the metric μ is supported on (or generated by) G. Recall that l_1-space is a set X of all real vectors equipped with the 1-norm $\| \cdot \|_1$, that is,

$$d_1(x, y) = \sum_{k=1}^{\infty} |x_k - y_k|$$

for all x, $y \in X$, where $x = (x_1, x_2, \cdots)$, $y = (y_1, y_2, \cdots)$.

Let (S, ρ) be another metric space. An *embedding* of G into (S, ρ) is a mapping $\varphi : V \to S$. The *distortion* of φ is the smallest value $c \geq 1$ such that

$$d_G(x, y) \leq \rho(\varphi(x), \varphi(y)) \leq c \cdot d_G(x, y) \;\; \forall x, y \in V.$$

Thus the distortion measures the maximum factor by which any distance is stretched in the embedding.

Problem 1 ([1]). *Is there is an absolute constant $C > 0$ so that every metric of a planar graph embeds into l_1-space with distortion $< C$?*

An *outerplanar* graph G is a planar graph with an embedding in the plane so that every vertex lies on the outer (unbounded) face. A *series-parallel* graph $G = (V, E)$ with terminals $s, t \in V$ is either a single edge (s, t), or a *series combination* or a *parallel combination* of two series parallel graphs G_1 and G_2 with terminals s_1, t_1 and s_2, t_2. The series combination of G_1 and G_2 is formed by setting $s = s_1, t = t_2$ and identifying $s_2 = t_1$; the parallel combination is formed by identifying $s = s_1 = s_2, t = t_1 = t_2$.

Known:

Theorem 2 ([2]). *Outer-planar graphs are isometrically embeddable into l_1.*

Theorem 3 ([2]). *There exists an embedding f of the $K_{2,n}$-metric into l_1 with distortion 3/2.*

Theorem 4 ([3]). *The class of series-parallel graphs embed into l_1 with distortion at most 16.*

References

[1] N. Linial, Finite metric spaces–combinatorics, geometry and algorithms, Proc. of the ICM (Beijing, 2002), 573-586.

[2] A. M. Charikar, R. Bhargava, Lower bounds on embeddings of planar graphs into the l_1 metric, Princeton University, (2004).

[3] A. Gupta, I. Newman, Y. Rabinovich, A. Sinclair, Cuts, trees, and l_1-embeddings of graphs, Combinatorica 24 (2) (2004) 233–269.

Problem

Proposed by Guanghui Wang, Shandong University

A rainbow subgraph of an edge-colored graph G is a subgraph of G where every edge has a distinct color.

Conjecture 1 (Schrijver)**.** *If G is a properly edge-colored d-regular graph, then G contains a rainbow path of length $d-1$.*

Problem

Proposed by Xiumei Wang, Zhengzhou University

Conjecture 1 (Fan-Raspaud Conjecture, 1994)**.** *In every 2-connected cubic graph, there exist three perfect matchings* M_1*,* M_2*, and* M_3 *such that* $M_1 \cap M_2 \cap M_3 = \emptyset$*.*

Song Xiaoxin[2] proved in his doctoral dissertation that for such 3-regular graphs, the conjecture holds: there exists a 2-factor with at most two odd cycles and at most three even cycles, or there exists a 2-factor with exactly four odd cycles and the graph is 4-edge-connected. Kaiser and Raspaud proved that the conjecture holds for 3-regular graphs with girth at most 4 [3]. Máčajová and Škoviera proved that the conjecture holds for 3-regular graphs that contain a 2-factor with at most two odd cycles [4]. Wang and Lin proved the conjecture for perfect matching polytopes of small dimensions [5], and studied the conjecture from the perspective of the core index of the perfect matching polytope of a 2-connected cubic graph [6]. Here, the *core index* $\varphi(P(G))$ of the polytope $P(G)$ is the minimum number of vertices of $P(G)$ whose convex hull contains $\left(\frac{1}{3}, \frac{1}{3}, \ldots, \frac{1}{3}\right)$.

References

[1] G. Fan and A. Raspaud, Fulkerson's conjecture and circuit covers, *J. Combin. Theory Ser. B* **61** (1994) 133–138.

[2] X. Song, Some Results on the Fulkerson Conjecture, Ph.D. Thesis, Chinese Academy of Sciences, Academy of Mathematics and Systems Science, Beijing, China (2002).

[3] T. Kaiser and A. Raspaud, Perfect matchings with restricted intersection in cubic graphs, *European J. Combin.* **31** (2010) 1307–1315.

[4] E. Máčajová and M. Škoviera, Sparsely intersecting perfect matchings in cubic graphs, *Combinatorica* **34** (2014) 61–94.

[5] X. Wang and Y. Lin, Three-Matching Intersection Conjecture for perfect matching polytopes of small dimensions, *Theoret. Comput. Sci.* **482** (2013) 111–114.

[6] X. Wang and Y. Lin, Core index of perfect matching polytope for a 2-connected cubic graph, *Discuss. Math. Graph Theory* **38** (2018) 189–201.

The problem on classifying ovoids in $W(3,q)$

Proposed by Qing Xiang, Southern University of Science and Technology

Let $V = \mathbb{F}_q^4$, where q is a prime power. We define a non-degenerate alternating form κ on V by setting

$$\kappa(x,y) = x_0y_3 + x_3y_0 + x_1y_2 + x_2y_1,$$

for $x = (x_0, x_1, x_2, x_3) \in V, y = (y_0, y_1, y_2, y_3) \in V$. Two vectors x, y are called *perpendicular* if $\kappa(x,y) = 0$. A subspace U is totally isotropic (t.i.) if any two vectors in U are perpendicular. It is known that $\dim(U) \leq 2$ if U is t.i.. Since κ is alternating, every 1-dim subspace of V is t.i.. The symplectic polar space $W(3,q)$ is the point-line incidence structure, where (i) the points are all the 1-dim subspaces of V, (ii) the lines are all the t.i. 2-dim subspaces of V. There are $(q+1)(q^2+1)$ points and $(q+1)(q^2+1)$ lines in $W(3,q)$, and $W(3,q)$ is a generalized quadrangle of order (q,q).

Let Γ be the collinearity graph of $W(3,q)$. That is, the vertices of Γ are the points of $W(3,q)$ and $\langle x \rangle \sim \langle y \rangle$ iff $\kappa(x,y) = 0$. The graph Γ is a strongly regular graph on $(q^2+1)(q+1)$ vertices. The maximum size of independent sets of Γ is q^2+1, and an independent set of maximum size is called an *ovoid* of $W(3,q)$. The ovoids of $W(3,q)$, with q odd, have been classified: they are all elliptic quadrics. So we will focus on the q even case. There are two known infinite families of ovoids when q is even: an elliptic quadric $Q^-(3,q) = \{\langle x \rangle \mid Q(x) = 0\}$ with $Q(x) = x_0x_3 + x_1^2 + ax_2^2 + x_1x_2$ ($\mathrm{Tr}_{\mathbb{F}_q/\mathbb{F}_2}(a) = 1$), a Suzuki ovoid for $q = 2^e$ with e odd. These known ovoids are usually called *classical*.

Conjecture. The classical ovoids mentioned above are all the ovoids in $W(3,q)$, where $q = 2^e$. In particular, if $q = 2^e$, where e is even, then the only ovoids in $W(3,q)$ are the elliptic quadrics.

References

[1] M. R. Brown. Ovoids of $PG(3,q)$, q even, with a conic section. *Journal of the London Mathematical Society. Second Series*, 62(2):569–582, 2000.

[2] H. Lüneburg. *Translation Planes.* Springer, Berlin, 1st edition edition, July 1980.

[3] T. Penttila and C. E. Praeger. Ovoids and translation ovals. *Journal of the London Mathematical Society*, 56(3):607–624, 1997.

[4] J. Tits. Ovoïdes et groupes de Suzuki. *Archiv der Mathematik*, 13:187–198, 1962.

Simple graph matrix whose spectrum determines the structure of random graphs

Proposed by Ziqing Xiang, Southern University of Science and Technology

In 2003, van Dam and Haemers asked the question whether the spectrum of some "sensible" graph matrix can *determine the structure of random graphs* (DSRG in short), where the meaning of "sensible" is part of the question.

There are numerical evidences suggesting the adjacency matrix, the adjacency matrix of the complement, the Laplacian matrix and the signless Laplacian matrix are DSRG.

Recently, it has been shown that some "natural" graph matrix is DSRG, where "natural" is defined rigorously in a specific way. This is more of an existence proof though, and the matrix constructed in the proof is quite complicated.

Since we know that some matrix is DSRG, it is very interesting to find simpler matrix. Ideally, the closer the matrix to the adjacency matrix, the better.

Canonical Ramsey property for L_3

Proposed by Zixiang Xu, IBS ECOPRO

Let L_s denote the s-point collinear configuration in $\mathbb{E}^d$ whose consecutive inter-point distances are all 1. In particular,

$$L_3 = \{x_1, x_2, x_3\} \subset \mathbb{E}^d \quad \text{with} \quad x_1, x_2, x_3 \text{ collinear and } |x_1x_2| = |x_2x_3| = 1.$$

Canonical Ramsey property. A finite configuration X is said to have the *canonical Ramsey property* if there exists $n_0(X)$ such that for every number of colors $r \in \mathbb{N}$ and every $n \geq n_0(X)$,

$$\mathbb{E}^n \xrightarrow{r} (X; X)_{\mathrm{GR}},$$

i.e., every r-coloring of $\mathbb{E}^n$ contains a monochromatic congruent copy of X or a rainbow congruent copy of X.

Problem 1. Does L_3 have the canonical Ramsey property? *Equivalently, is there an n_0 such that for every $r \in \mathbb{N}$ and every $n \geq n_0$,*

$$\mathbb{E}^n \xrightarrow{r} (L_3; L_3)_{\mathrm{GR}}\,?$$

Remarks. (1) The answer for L_s with $s \geq 4$ is NO. (2) If we additionally require the coloring function is spherical, then the answer is YES.

2 problems from Coding Theory and Extremal Set Theory

Proposed by Guiying Yan, Academy of Mathematics and Systems Science

The maximum size of a q-ary code of length n with a minimum Hamming distance of d, denoted as $A_q(n, d)$, can be framed as an independence number problem on a specific graph known as the Gilbert graph.

This graph is defined as follows:

- Its vertex set consists of all q-ary vectors of length n.
- An edge exists between any two vertices if and only if their Hamming distance is greater than 0 but less than d.

Consequently, the Gilbert-Varshamov bound is a natural and direct result derived from finding a lower bound on the independence number of this graph.

Theorem 1 (Gilbert-Varshamov bound, [3])**.**

$$A_q(n, d) \geq \frac{q^n}{\sum_{i=0}^{d} \binom{n}{i} (q-1)^i}$$

However, a significant gap remains between the known upper and lower bounds for $A_q(n, d)$, even in the asymptotic case.

Problem 2. *Calculate $A_q(n, d)$ for all q, n and d, and determine specific codes that achieve this maximum size.*

The problem of finding the maximum number of codewords is related to the celebrated Erdős–Ko–Rado Theorem.

Theorem 3 (Erdős–Ko–Rado Theorem, [2])**.** *Suppose that $\mathcal{F}$ is a family of distinct k-element subsets of an n-element set with $1 \leq k \leq n/2$, and that each two subsets share at least one element, then*

$$|\mathcal{F}| \leq \binom{n-1}{k-1}.$$

According to a result from [1], the maximum size of a k-intersecting set of permutations of length n is at most $(n-k)!$, provided that n is sufficiently large relative to k.

In the context of binary codes, a codeword can be seen as an indicator vector for a subset. Consequently, the Hamming distance between two codewords is equivalent to the size of the symmetric difference between their corresponding subsets.

Problem 4. *What is the maximum size of a family of distinct subsets of an n-element set, such that the symmetric difference of any two subsets in the family is greater than d?*

References

[1] D. Ellis, E. Friedgut, H. Pilpel, Intersecting families of permutations. *Journal of the American Mathematical Society*, 2011, 24(3): 649–682.

[2] P. Erdős, C. Ko and R. Rado, Intersection theorems for systems of finite sets, *Quart. J. Math. Oxford*, (2) 12 (1961), pp. 313–320.

[3] E. N. Gilbert. A comparison of signalling alphabets, *The Bell system technical journal*, 31(3):504–522, 1952.

Open problems of induced subdivisions

Proposed by Fan Yang, Shandong University & ECOPRO

If G, H are graphs, G is *H-free* if no induced subgraph of G is isomorphic to H. For a graph H, a *subdivision* of H, is a graph obtained by replacing edges of H by internally vertex-disjoint paths. We say G is *H-subdivision-free* if no induced subgraph of G is isomorphic to a subdivision of H.

Conjecture 1 (Gyárfás-Sumner[1, 5])**.** *For every forest H and every integer $k \geq 1$, there exists $c \geq 1$ such that $\chi(G) \leq c$ for every $\{H, K_{k+1}\}$-free graph G.*

The Conjecture 1 is nearly true in the following sense:

Theorem 2 ([3])**.** *For every forest H and integer k, every $\{H, K_{k+1}\}$-free graph G satisfies $\chi(G) \leq |G|^{o(1)}$.*

The Conjecture 1 is true for replacing H-free by H-subdivision-free.

Theorem 3 ([4])**.** *For every forest H and every integer $k \geq 1$, there exists $c \geq 1$ such that every K_{k+1}-free, H-subdivision-free graph G satisfies $\chi(G) \leq c$.*

However, Scott [4] suggested an analogue of Conjecture 1 that we could replace the exclusion of a forest with all subdivisions of any given graph.

Conjecture 4 (False)**.** *For every graph H, and every integer $k \geq 1$, there exists c such that every K_{k+1}-free, H-subdivision-free graph G satisfies $\chi(G) \leq c$.*

Theorem 5 gives a counterexample to Conjecture 4.

Theorem 5 ([6])**.** *Let H be obtained from K_4 by subdividing once every edge in a cycle of length 4. There are infinitely many K_3-free and H-subdivision-free graphs G such that $\alpha(G) < |G|/\log\log|G|$.*

Nevertheless, a near-optimal lower bound on the independent number still holds.

Theorem 6 ([2])**.** *For all graphs H and integers $k \geq 1$, every H-subdivision-free and K_{k+1}-free graph G satisfies*

$$\alpha(G) \geq \frac{|G|}{poly \log |G|}.$$

Problem 7. *Can the bound in Theorem 6 be improved to*

$$\alpha(G) \geq \frac{|G|}{poly \log\log |G|}.$$

References

[1] A. Gyárfás, On ramsey covering-numbers, Coll. Math. Soc. János Bolyai, in Infinite and Finite Sets, 1975, 10.

[2] T. Nguyen, A. Scott, P. Seymour, Subdivisions and near-linear stable sets, arXiv:2409.09400v1, 2024.

[3] T. Nguyen, A. Scott, P. Seymour, Trees and near-linear stable sets, arXiv:2409.09397v2, 2025.

[4] A. Scott, Induced trees in graphs of large chromatic number, J. Graph Theory, 1997, 297-311.

[5] D. P. Sumner, Subtrees of a graph and chromatic number, in The Theory and Applications of Graphs, 1981, 557-576.

[6] B. Walczak, Triangle-free geometric intersection graphs with no large independent set, Discrete and Combinatorial Geometry, 2015, 221-225.

Acute polytopes

Proposed by Liping Yuan, Hebei Normal University

Let $P \subset \mathbb{R}^d$ be a polytope with vertices $a_1, a_2, \cdots, a_n$. Any angle $\widehat{a_i a_j a_k}$ is called an *angle at* a_j of P, if $a_i a_j a_k$ lies in a facet of P, and $a_i a_j, a_j a_k$ are edges of P.

A polytope is called *acute*, if it has an *acute vertex* v, which means that the angles at v of all 2-dimensional faces containing v are acute. Interestingly, every tetrahedron is acute.

Let $V(P)$ be the vertex set of the polytope P. By removing a vertex $v \in V(P)$, we obtain the polytope $\mathrm{conv}(V(P) \setminus \{v\})$. Suppose card $V(P) = n$. By successively taking off m vertices from $V(P)$ in a suitable manner, we finally obtain, for $m = n - 4$, a tetrahedron.

The minimal m, for which we obtain an acute polytope is the *obtuseness* of P. If $m = 0$, then P is acute; if $m = n - 4$, then P is called *obtuse*. For example, the cube is neither acute nor obtuse, and its obtuseness is 2.

Problem 1. *Determine the obtuseness m of a polytope with n vertices.*

Forcing numbers of perfect matchings of hypercubes

Proposed by Heping Zhang, Lanzhou University

For a perfect matching M of a graph G, a *forcing set* of M in G is an edge subset $S \subseteq M$ such that S is contained in no other perfect matchings of G, and the *forcing number* of M is the smallest cardinality of a forcing set of M, denoted by $f(G, M)$. Let $f(G) = \min\{f(G, M) : M \in \mathcal{M}(G)\}$, $F(G) = \max\{f(G, M) : M \in \mathcal{M}(G)\}$ and $\text{Spec}(G) = \{f(G, M) : M \in \mathcal{M}(G)\}$, where $\mathcal{M}(G)$ denotes the set of perfect matchings of G. Then $f(G)$, $F(G)$ and $\text{Spec}(G)$ are called the minimum forcing number, maximum forcing number and forcing spectrum of G respectively. An n-cube Q_n is n-fold Cartesian product of K_2.

For the minimum forcing number of Q_n, in 2019 Diwan [1] proposed a linear algebra method to obtain $f(Q_n) = 2^{n-2}$ for $n \geq 2$, settling a conjecture due to Pachter and Kim (1998) that was revised and confirmed in even n by Riddle. For the maximum forcing number, Alon showed $F(Q_n) > c2^{n-1}$ for any constant $0 < c < 1$ and sufficiently large n. Using Alon's method, Adams et al. [2] obtained a lower bound in a more general sense as follows.

$$F(Q_n) \geq (1 - \frac{\log(2e)}{\log n})2^{n-1}.$$

Recently Zhang et al. [3] pointed out an upper bound on $F(Q_n)$ from a result Diwan just proved.

$$F(Q_n) \leq (1 - \frac{1}{n})2^{n-1}.$$

Problem 1. *Give an explicit or asymptotic expression for the maximum forcing number $F(Q_n)$.*

Problem 2. *Determine the forcing spectrum* $\text{Spec}(Q_n)$.

There are some progresses [2] on Problem 2: $\text{Spec}(Q_n) = \{2^{n-2}\}$ for $2 \leq n \leq 4$, $\text{Spec}(Q_5) = \{8, 9\}$ and $\text{Spec}(Q_n) \supseteq \{2^{n-2}, 2^{n-2} + 1, \ldots, 2^{n-2} + 2^{n-5}\}$ for $n \geq 5$. For $n \geq 6$, the problem remains open. It is desirable to show that $\text{Spec}(Q_n)$ forms an integer interval.

References

[1] A. A. Diwan, The minimum forcing number of perfect matchings in the hypercube, Discrete Math. 342 (2019) 1060-1062.

[2] P. Adams, M. Mahdian, E. S. Mahmoodian, On the forced matching numbers of bipartite graphs, Discrete Math. 281 (2004) 1-12.

[3] Y. Zhang, X. He, Q. Liu, H. Zhang, Forcing, anti-forcing, global forcing and complete forcing on perfect matchings of graphs – A survey, Discrete Appl. Math. 376 (2025) 318-347.

On maximizing algebraic connectivity

Proposed by Shenggui Zhang, Northwestern Polytechnical University

For a graph, the second smallest Laplacian eigenvalue is called its algebraic connectivity [1]. As an important parameter in spectral graph theory, the algebraic connectivity of a graph reflects its connectivity and offers a spectral perspective on classical problems in graph theory, including the isoperimetric number [2] and the expansion of graphs [3]. Moreover, it has received great attention in various fields including complex networks, multi-agents systems, and so on. A larger algebraic connectivity implies stronger expansion of a graph, stronger robustness in a complex network, as well as a faster consensus convergence rate of a multi-agent system. We are interested in determining the maximum algebraic connectivity and the corresponding graphs among some specific classes of graphs. However, it is NP-hard to find a graph that maximizes the algebraic connectivity among all graphs of given order and size [4]. The graphs maximizing the algebraic connectivity among the graphs of given order n and size $n-1, n, n+1, n+s, t(t+1)$ were determined respectively, where $5s+4 \leq n$ and $t - \frac{2t^2}{n} < 1$ [5, 6, 7].

Problem 1. *For which classes of graphs with given order n and size m, where m is a function of n, can the maximum algebraic connectivity and the corresponding extremal graphs be determined?*

Problem 2. *For graphs with given order n and size m, can we determine which edge (or set of edges) will lead to the maximum increase in algebraic connectivity under one type of graph operation once?*

Problem 3. *For graphs with certain structural properties, can we determine the maximum algebraic connectivity and the corresponding extremal graphs?*

References

[1] M. Fiedler, Algebraic connectivity of graphs, *Czechoslovak Math. J.*, **23** (98) (1973) 298–305.

[2] B. Mohar, Isoperimetric numbers of graphs, *J. Combin. Theory Ser. B*, **47** (3) (1989) 274–291.

[3] S. Hoory, N. Linial, and A. Wigderson, Expander graphs and their applications, *Bull. Amer. Math. Soc. (N.S.)*, **43** (4) (2006) 439–561.

[4] D. Mosk-Aoyama, Maximum algebraic connectivity augmentation is NP-hard, *Oper. Res. Lett.*, **36** (2008) 677–679.

[5] R. Grone and R. Merris, Ordering trees by algebraic connectivity, *Graphs Combin.* **6** (1990) 229–237.

[6] A. Lal, K. Patra, and B. Sahoo, Algebraic connectivity of connected graphs with fixed number of pendant vertices, *Graphs Combin.*, **27** (2011) 215–229.

[7] K. Ogiwara, T. Fukami, and N. Takahashi, Maximizing algebraic connectivity in the space of graphs with fixed number of vertices and edges, *IEEE Trans. Control Netw. Syst.*, **4** (2) (2017) 359–368.

Bounds on Separating Hash Families

Proposed by Xiande Zhang, University of Science and Technology of China

An $(n \times m)$-matrix over $\mathbb{F}_q$ is called a *separating hash family* of type $\{w_1, \ldots, w_t\}$, denoted by $SHF(n; m, q, \{w_1, \ldots, w_t\})$, if for all pairwise disjoint subsets $C_1, \ldots, C_t$ of columns with $|C_i| = w_i \geq 1$ for $1 \leq i \leq t$, there exists at least one row that *separates* $C_1, \ldots, C_t$, that is, in this row, the t sets of elements in columns from C_i, $i \in [t]$, are pairwise disjoint.

Given integers n, q and $w_1, \ldots, w_t$, denote the maximum number of columns m as $C(n, q, \{w_1, \ldots, w_t\})$, such that there exists an $SHF(n; m, q, \{w_1, \ldots, w_t\})$. It is known that

$$\Omega_{n,u}(q^{\frac{n}{u-1}}) = C(n, q, \{w_1, \ldots, w_t\}) \leq (u-1)q^{\lceil \frac{n}{u-1} \rceil},$$

where $u := \sum_{i=1}^{t} w_i$. This means that when $(u-1) \mid n$, the exponent of q has been determined.

Problem 1 (Blackburn et al. 2008)**.** *Let n and w_i be fixed positive integers. If $(u-1) \nmid n$, then for sufficiently large q and arbitrarily small $\epsilon > 0$, does there exist an $SHF(n; m, q, \{w_1, \ldots, w_t\})$ such that $m \geq q^{\lceil n/(u-1) \rceil - \epsilon}$?*

Recently, with Wei and Ge, we positively answered this question for $n = u$. It is still widely open for $n > u$. For $t = 2$, the smallest open case is $C(6, q, \{2, 3\})$.

Open problems on total positivity

Proposed by Bao-Xuan Zhu, Jiangsu Normal University

A matrix of real numbers is *totally positive* if all its minors are nonnegative. Total positivity of matrices has been deeply studied and plays an important role in various branches of mathematics [1], such as classical analysis, representation theory, network analysis, cluster algebra, positive Grassmannians and integrable systems, combinatorics.

Let S_n denote the set of all permutations on the set $[n] := \{1, 2, \ldots, n\}$. We say that a permutation $\pi \in S_n$ has an *ascent* at the position i if $\pi_i < \pi_{i+1}$, where $i \in [n-1]$. The *Eulerian number* $\left\langle {n \atop k} \right\rangle$ is the number of all permutations in S_n having k ascents. It is well known that $\left\langle {n \atop k} \right\rangle$ satisfies the recurrence relation

$$\left\langle {n \atop k} \right\rangle = (k+1)\left\langle {n-1 \atop k} \right\rangle + (n-k)\left\langle {n-1 \atop k-1} \right\rangle, \tag{1}$$

where $\left\langle {0 \atop 0} \right\rangle = 1$ and $\left\langle {n \atop k} \right\rangle = 0$ unless $0 \leq k < n$ (see [2, A008292] for details). The Eulerian triangle

$$\left[\left\langle {n \atop k} \right\rangle\right]_{n,k\geq 0} = \begin{bmatrix} 1 & & & & & \\ 1 & 1 & & & & \\ 1 & 4 & 1 & & & \\ 1 & 11 & 11 & 1 & & \\ 1 & 26 & 66 & 26 & 1 & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}.$$

A major longstanding open problem in the theory of total positivity is as follows.

Conjecture 1 ([3], Conjecture 6.10)**.** *The Eulerian triangle* $\left[\left\langle {n \atop k} \right\rangle\right]_{n,k\geq 0}$ *is totally positive.*

The generalized Eulerian number $\left\langle {n \atop k} \right\rangle^{(r,s)}$ was defined to satisfy the recurrence relation

$$\left\langle {n \atop k} \right\rangle^{(r,s)} = [s(n-1)+k+1]\left\langle {n-1 \atop k} \right\rangle^{(r,s)} + [r(n-1)-k+1]\left\langle {n-1 \atop k-1} \right\rangle^{(r,s)}, \tag{2}$$

where $\left\langle {0 \atop 0} \right\rangle^{(r,s)} = 1$ and $\left\langle {n \atop k} \right\rangle^{(r,s)} = 0$ unless $0 \leq k \leq n-1$.

The generalized Eulerian number is a common generalization of the classical Eulerian number, the rth-order Eulerian number and the degenerate Eulerian number. We conjecture:

Conjecture 2 ([4])**.** *For any* $r, s \in \mathbb{N}$*, the triangle* $[\left\langle {n \atop k} \right\rangle^{(r,s)}]_{n,k\geq 0}$ *is totally positive.*

References

[1] S. Karlin, Total Positivity, Vol. I, Stanford University Press, Stanford, 1968.

[2] N. J. A. Sloane, The on-line encyclopedia of integer sequences, http://oeis.org.

[3] F. Brenti, Combinatorics and total positivity, J. Combin. Theory A 71 (1995) 175–218.

[4] M.-J. Ding, L. L. Mu, B.-X. Zhu, Stieltjes moment property of the generalized Eulerian polynomials, submitted to Adv. in Appl. Math..

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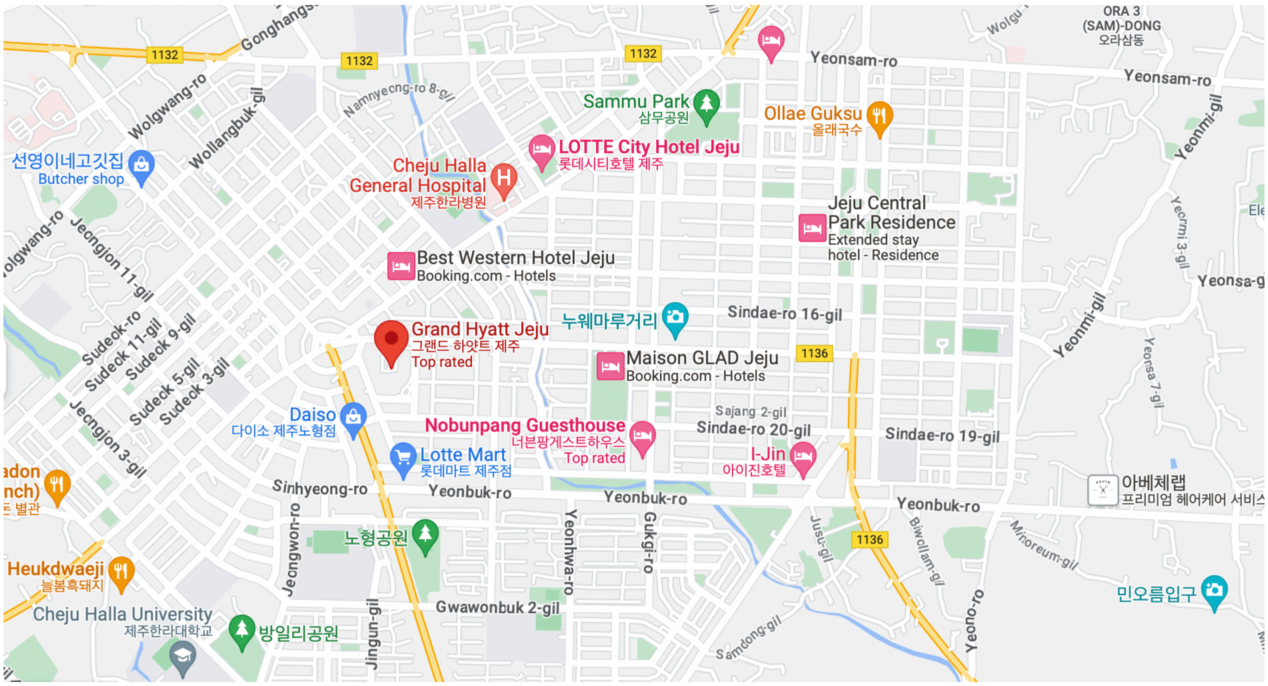
How to get from the airport to Grand Hyatt Jeju

Grand Hyatt Jeju is located 4km from Jeju International airport: 10 minute by car; 6km from Jeju International ferry terminal: 15 minute by car. Duty free shops, local markets, shopping streets, including restaurants and bars are located within close proximity.



Grand Hyatt Jeju
12 Noyeon-ro, Jeju-si, South Korea, 63082.
(+82) 64 907 1234

Public Taxi
An approximately 10-minute journey from Jeju International Airport
Hotel address to provide taxi driver: 제주시 노연로 12 그랜드 하얏트 제주

Public Bus
Bus stop location at Jeju International Airport: No 6
(in-front of Gate 5-International Arrival)
Bus fare (cash): Adult 1,200 KRW/ Child 400 KRW
Bus Stop near hotel: Alight at Wonnohyeong (approx. 2 minutes walk):
Bus Service Numbers:
Bus No 316 – Alight on the 7th stop from airport
Bus No 465 – Alight on the 7th stop from airport
Bus No 365 – Alight on the 11th stop from airport

*Bus route and information can be changed per operation.
Please visit here (http://bus.jeju.go.kr/?lang=en) for more information.