2023 Developments in Combinatorics Workshop

Grand Hyatt Jeju

Jeju Island, South Korea

October 20-23, 2023

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/development-in-combinatorics-3rd-workshop-2023/

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Organizing Committee

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Sponsored by

Institute for Basic Science

Timetable

- October 20, Friday
  - 15:00 – 18:00 Registration & Free discussion
- October 21, Saturday
  - 08:00 – 09:00 Registration (Room 4 or 5 on the fourth floor)
  - Session 21A
    - 09:00 – 09:15 Talk 21A-1
    - 09:15 – 09:30 Talk 21A-2
    - 09:30 – 09:45 Talk 21A-3
    - 09:45 – 10:00 Talk 21A-4
    - 10:00 – 10:15 Talk 21A-5
    - 10:15 – 10:30 Talk 21A-6
  - 10:30 – 11:00 **Coffee Break**
  - Session 21B
    - 11:00 – 11:15 Talk 21B-1
    - 11:15 – 11:30 Talk 21B-2
    - 11:30 – 11:45 Talk 21B-3
    - 11:45 – 12:00 Talk 21B-4
  - 12:00 – 14:00 **Lunch**
  - Session 21C
    - 14:00 – 14:15 Talk 21C-1
    - 14:15 – 14:30 Talk 21C-2
    - 14:30 – 14:45 Talk 21C-3
    - 14:45 – 15:00 Talk 21C-4
  - Session 21D
    - 15:00 – 17:00 Free discussion
  - 18:00 – 20:00 **Banquet**

    (Noknamu (녹나무) restaurant on the third floor of Grand Hyatt Jeju)

- October 22, Sunday
  - Session 22A
    - 09:00 – 09:15 Talk 22A-1
    - 09:15 – 09:30 Talk 22A-2
    - 09:30 – 09:45 Talk 22A-3
    - 09:45 – 10:00 Talk 22A-4
    - 10:00 – 10:15 Talk 22A-5
    - 10:15 – 10:30 Talk 22A-6
  - 10:30 – 11:00 **Coffee Break**
  - Session 22B
    - 11:00 – 11:15 Talk 22B-1
    - 11:15 – 11:30 Talk 22B-2
    - 11:30 – 11:45 Talk 22B-3
    - 11:45 – 12:00 Talk 22B-4
  - 12:00 – 14:00 **Lunch**
  - Session 22C
    - 14:00 – 14:15 Talk 22C-1
    - 14:15 – 14:30 Talk 22C-2
    - 14:30 – 14:45 Talk 22C-3
    - 14:45 – 15:00 Talk 22C-4
  - Session 22D
    - 15:00 – 17:00 Free discussion
- October 23, Monday
  - Session 23A
    - 09:00 – 12:00 Free discussion

Graphs of large twin-width

Proposed by Jungho Ahn, KIAS

Problem: For a positive integer n, which n-vertex graph attains the largest twin-width?

A new graph parameter, called twin-width, was recently introduced and plays important roles in both computer science and structural graph theory. Its definition uses the following operation, which is one generalization of the graph minor. For distinct vertices v and w of a graph G, we denote by $G/\{u, v\}$ the graph G' (possibly with red edges) obtained from $G - \{u, v\}$ by adding a vertex x such that for every vertex $w \in V(G) \setminus \{u, v\}$, the following hold:

- if both u and v are joined to w by black edges in G, then x and w are joined by a black edge in G',
- if both u and v are non-adjacent to w in G, then x and w are non-adjacent in G', and
- otherwise, x and w are joined by a red edge in G'.

We call this operation *contracting u and v*. A *contraction sequence* of G is a sequence of graphs from G to a 1-vertex graph obtained by repeatedly contracting vertices. The *width* of the sequence $G_1(= G), \ldots, G_t$ is the maximum red degree of G_i among all $i \in [t]$, where red degree counts the number of red edges incident with a vertex. The *twin-width* of G is the minimum width of a contraction sequence among all possible contraction sequences.

Currently, Paley graphs are known to attain the largest twin-width, $(n-1)/2$. We wonder whether there exists an n-vertex graph of twin-width larger than $(n-1)/2$, or the largest twin-width is attained by strongly regular graphs like Paley graphs.

Parial line transversals for pairwise intersecting convex sets in $\mathbb{R}^3$

Proposed by Minho Cho, IBS ECOPRO

Let F be a family of sets on a ground set V. F is called *pairwise intersecting* if $C_1 \cap C_2 \neq \emptyset$ for every $C_1, C_2 \in F$. A subset $T \subseteq V$ is a *transversal* for the family F if $T \cap C \neq \emptyset$ for every $C \in F$.

Conjecture. *There exists an absolute constant $\epsilon > 0$ which satisfies the following: for every finite pairwise intersecting family F of convex sets in $\mathbb{R}^3$, there exists a subfamily $F' \subseteq F$ of size $|F'| \geq \epsilon|F|$ which admits a line transversal.*

Known: the conjecture is true if F is a family of cylinders. (A *cylinder* is the Minkowski sum of a convex set and a line in $\mathbb{R}^3$.)

Question. *Is the conjecture true if F is a family of half-cylinders?*

Question. *Is the conjecture true if F is a family of three-dimensional simplices?*

References

[1] I. Bárány, Pairwise intersecting convex sets and cylinders in $\mathbb{R}^3$, Preprint, arXiv: 2104.02148.

[2] L. Martínez-Sandoval, E. Roldán-Pensado and N. Rubin, Further consequences of the colorful Helly hypothesis, *Discrete Comput. Geom.*, 63 (2020), 848–866.

Progress on Hippchen's Conjecture

Proposed by Ilkyoo Choi, Hankuk University of Foreign Studies

Hippchen [3] conjectured that two longest paths in a k-connected graph share at least k vertices. For each k, there are infinitely many graphs for which the conjecture, if true, is tight. (The complete bipartite graph $K_{k,2k+2}$ is an example.) The conjecture is known to be true when $k \leq 5$ or $k \geq \frac{n+2}{5}$ [2].

On the other hand, two longest cycles in a k-connected graph share at least $ck^{3/5}$ vertices [1], where $c \approx 0.2615$. This can be used to prove that every two longest paths in a k-connected graph have at least common $ck^{3/5}$ vertices.

Any improvements would be interesting.

References

[1] Guantao Chen, Ralph J. Faudree, and Ronald J. Gould, Intersections of longest cycles in k-connected graphs, *J. Combin. Theory Ser. B*, 72(1):143–149, 1998.

[2] Eun-Kyung Cho, Ilkyoo Choi, Boram Park, Improvements on Hippchen's conjecture, *Discrete Mathematics*, 345(11):113029, 2022.

[3] Thomas Hippchen, Intersections of longest paths and cycles, *Thesis, Georgia State University*, 2008.

Problem

Proposed by Jeong Ok Choi, GIST

Definition. *On* $[n] = \{0, 1, \cdots, n-1\}$*, let* $S_i = 2^{[n]-\{i\}}$ *and* $s \in S_0 \times S_1 \times \cdots \times S_{n-1}$*. Consider* $s = (s_0, s_1, \cdots, s_{n-1})$*. The graph* $G[s]$ *on* $[n]$ *is the graph on* $[n]$ *with the edge set* $\bigcup_{i=0}^{n-1}(\{i\} \times s_i)$*. The cost by vertex* i *under* s *is*

$$c_i(s) = \alpha|s_i| + \sum_{i,j} d_{G[s]}(i,j) \quad \textit{in} \quad G[s].$$

Definition. *A Nash Equilibrium is an* s *(and* $G[s]$ *as well) such that for each vertex* i*,* $c_i(s) \geq c_i(s')$ *whenever* s' *differs from* s *in the* i*th component.*

Conjecture (Fabrikant et al. 2003)**.** *There is a constant* K *such that every NE is a tree if* $\alpha > K$*.*

Known by Albers et al. The conjecture is false.

Revised Conjecture: Every NE is a tree if $\alpha \geq n$.

Theorem (Dippel and Vetta, 2022)**.** *If* $\alpha > 3(n-1)$*, then every NE is a tree.*

Problem. *Improve the result or prove/disprove the revised conjecture.*

References

[1] S. Albers, S. Eilts, E. Even-Dar, Y.Mansour, and L. Roditty, On Nash equilibria for a network creation game, *ACM Transactions on Economics and Computiation*, 2(1), 2014.

[2] A. Fabrikant, A. Luthra, E. Maneva, C. papadimitriou, and S. SHenker, On a network creation game, *Proceedings of 22nd Symposium on Priciples of Distributed Computing (PODC)*, pages 347-351, 2003.

[3] J. Dippel and A. Vetta, An Improved Bound for the Tree Conjecture in Network Creation Games, *Algorithmic Game Theory. SAGT 2022. Lecture Notes in Computer Science*, vol. 13584, Springer, Cham.

Problem

Proposed by Alexander Clifton, IBS DIMAG

We define an essential k-cover of $Q^n := \{0,1\}^n$ as a collection of affine hyperplanes such that:

(i) Every point of Q^n is contained in at least k hyperplanes,

(ii) No hyperplane can be removed while retaining property (i), and

(iii) Every variable $x_1, \ldots, x_n$ is used with a nonzero coefficient in the defining equations of at least k hyperplanes.

We let $e(n,k)$ denote the minimum cardinality of an essential k-cover of Q^n. It is known that $e(n,k) = C_n k + O_n(1)$ where the constants C_n have been determined.

Question. Can we improve the bounds on the $O_n(1)$ term?

Question. Can we determine whether $e(n,k)$ is increasing in k and nondecreasing in n?

For the original problem where $k = 1$, see Linial–Radhakrishnan (2005) and Araujo–Balogh–Mattos (2022): **https://arxiv.org/abs/2209.00140**

References

[1] Nathan Linial, Jaikumar Radhakrishnan, Essential covers of the cube by hyperplanes, *Journal of Combinatorial Theory, Series A* 109(2): 331-338 (2005)

p-small and weakly p-small

Proposed by Jie Han, Beijing Institute of Technology

Given a finite set X, a family $\mathcal{F} \subseteq 2^X$ is called *increasing* if $B \supseteq A \in \mathcal{F} \Rightarrow B \in \mathcal{F}$. For a given X and $p \in [0,1]$, μ_p is the product measure on 2^X given by $\mu_p(S) := p^{|S|}(1-p)^{|X \setminus S|}$ for any $S \subseteq X$. For an increasing $\mathcal{F}$, the *threshold* $p_c(\mathcal{F})$ is the unique p for which $\mu_p(\mathcal{F}) = \frac{1}{2}$. We say $\mathcal{F}$ is *p-small* if there is a $\mathcal{G} \subseteq 2^X$ such that $\mathcal{F} \subseteq \langle \mathcal{G} \rangle := \{T : \exists S \in \mathcal{G}, S \subseteq T\}$ and $\sum_{S \in \mathcal{G}} p^{|S|} \leq \frac{1}{2}$. Then $q(\mathcal{F}) := \max\{p : \mathcal{F} \text{ is } p\text{-small}\}$, which is defined as the *expectation-threshold of $\mathcal{F}$*. Let $\ell(\mathcal{F})$ be the maximum size of minimal members of $\mathcal{F}$.

Talagrand [4] introduced the following LP relaxation of "p-smallness": we say $\mathcal{F}$ is *weakly p-small* if there is a function $g : 2^X \to \mathbb{R}^+$ such that

$$\sum_{S \subseteq T} g(S) \geq 1 \ \forall T \in \mathcal{F} \quad \text{and} \quad \sum_{S \subseteq X} g(S) p^{|S|} \leq \frac{1}{2}.$$

Then the *fractional expectation-threshold* for $\mathcal{F}$, denoted by $q_f(\mathcal{F})$, is defined as:

$$q_f(\mathcal{F}) := \max\{p : \mathcal{F} \text{ is weakly } p\text{-small}\}.$$

We indeed have $q(\mathcal{F}) \leq q_f(\mathcal{F}) \leq p_c(\mathcal{F})$ for increasing $\mathcal{F}$ (the second inequality was explained in [1]). Talagrand raised the following conjecture and referred it as a "very nice problem in Combinatorics".

Conjecture ([4])**.** *There exists a universal constant K such that for every increasing $\mathcal{F}$, we have $q(\mathcal{F}) \geq q_f(\mathcal{F})/K$.*

Frankston, Kahn, Narayanan, and Park [1] resolved a conjecture of Talagrand[4], which is a fractional relaxation of a conjecture of Kahn and Kalai. Very recently Park and Pham [3] resolved the Kahn-Kalai [2] conjecture.

Theorem ([1, 3])**.** *There exists a constant K such that for any finite X and increasing family $\mathcal{F} \subseteq 2^X$, we have*

$$q(\mathcal{F}) \leq q_f(\mathcal{F}) \leq p_c(\mathcal{F}) \leq Kq(\mathcal{F}) \log \ell(\mathcal{F}) \leq Kq(\mathcal{F}) \log \ell(\mathcal{F}).$$

References

[1] K. Frankston, J. Kahn, B. Narayanan, and J. Park, Thresholds versus fractional expectation thresholds, *Annals of Mathematics* 194 (2021) 475–495.

[2] J. Kahn and G. Kalai, Thresholds and expectation thresholds, *Combin. Probab. Comput.* 16(3)(2007), 495–502.

[3] J. Park and H. T. Pham, A proof of the Kahn-Kalai conjecture, *J. Amer. Math. Soc.* electronically published on August 7, 2023, DOI: https://doi.org/10.1090/jams/1028 (to appear in print).

[4] M. Talagrand, Are many small sets explicitly small? *Proceedings of the 2010 ACM International Symposium on Theory of Computing* (2010) 13–35.

Problem

Proposed by Cheolwon Heo, Korea Institute for Advanced Study

For binary matroids M and N, a *matroid homomorphism* $\phi : M \to N$ is a map $\phi : E(M) \to E(N)$ such that for every circuit C of M, $\phi(C)$ is a disjoint union of circuits of N. The *N-recolouring graph $RG(M,N)$ of M* is the graph whose vertex set is the set of matroid homomorphisms from M to N, in which two vertices ϕ and ϕ' are adjacent if for some cocircuit C of M, $\phi(e) \neq \phi'(e)$ if and only if $e \in C$. Then the *N-recolouring problem*, $RP(N)$, is defined as follows:

- Instance: A binary matroid M and two maps $\phi, \psi \in RG(M,N)$.
- Decision: Is there a path in $RG(M,N)$ between ϕ and ψ?

This problem is clearly in the complexity class $PSPACE$, and we expect that it will be in P or $PSPACE$-complete depending on N.

Theorem. *For $n \geq 2$ and $N = M(K_{2^n})$, $RP(N)$ is in $PSPACE$-complete.*

Conjecture. *For $n \geq 3$ and $N = M(K_n)$, $RP(N)$ is in $PSPACE$-complete.*

Problem

Proposed by Ping Hu, Sun Yat-sen University

The minimum color degree of an edge-colored graph G, denoted by $\delta^c(G)$, is the minimum number of colors assigned to the edges incident to a vertex of G. In 2013, Li proved that an edge-colored graph G of order n contains a rainbow triangle if $\delta^c(G) \geq (n+1)/2$. Let $TT^c(n,k)$ be the minimum number m such that if G is an edge-colored graph of order n with $\delta^c(G) \geq m$, then G contains k vertex-disjoint rainbow triangles. In 2020, Hu, Li, Yang conjectured that for all positive integers n and k with $n \geq 3k$, $TT^c(n,k) = (n+k)/2$.

We consider the asymptotic version of this question. Let $f(x) = \lim_{n\to\infty} TT^c(n, xn/3)/n$. Is it true that $f(x) = (3+x)/6$?

References

[1] Hao Li, Rainbow $C_3{}'$ s and $C_4{}'$ s in edge-colored graphs, *Discrete Mathematics*, Volume 313, Issue 19,2013, 1893-1896.

[2] Jie Hu, Hao Li, Donglei Yang, Vertex-disjoint rainbow triangles in edge-colored graphs, *Discrete Mathematics*, Volume 343, Issue 12,2020, 112117.

Algorithmic aspect of rainbow Dirac's theorem

Proposed by Seonghyuk Im, KAIST & IBS ECOPRO

A classical theorem by Dirac states that $\delta(G) \geq n/2$ implies the existence of Hamilton cycle. In fact, the proof of Dirac's theorem provide a (deterministic) polynomial-time algorithm that find a Hamilton cycle when $\delta(G) \geq n/2$ while finding a Hamilton cycle in a general graph is NP-hard. One may ask that whether we weaking the condition of the minimum degree, is there a fast algorithm to decide the existence of Hamilton cycle. Jansen, Kozma, and Nederlof proved that there exists $(30^{6k}n^3)$-time algorithm that decide Hamiltonicity of given G with $\delta(G) \geq n/2 - k$. Thus, this problem is in FPT. On the other hand, for fixed $\varepsilon > 0$, deciding Hamiltonicity of given G with $\delta(G) \geq (\frac{1}{2} - \varepsilon)n$ remains NP-complete. Joos and Kim improve the result of Dirac into the following "rainbow" setting.

Theorem. *Let $G_1, G_2, \ldots, G_n$ be graphs on a common vertex set V of size n and $\delta(G_i) \geq n/2$ for each $i \in [n]$. Then there exists a Hamilton cycle C in the complete graph on V and a bijection $\varphi : E(C) \to [n]$ such that $e \in E(G_{\varphi(e)})$ for every $e \in E(C)$.*

Question. *Can we decide the existence of "rainbow Hamilton cycle" efficiently when $\delta(G_i) \geq n/2 - k$ for each $i \in [n]$? If not, is it possible when only constantly many G_i has smaller minimum degree?*

Problem

Proposed by Donggyu Kim, KAIST & IBS DIMAG

A graph is bipartite if and only if it has no K_3-pivot-minor. So, it is natural to ask the structure of graphs having no K_4-pivot-minor. Davies announced that for any graph H, the class of H-pivot-minor-free graphs is χ-bounded. Hence the class of K_4-pivot-minor-free has bounded chromatic number. Note that W_5 has no K_4-pivot-minor and has chromatic number 4.

What is the correct bound of the chromatic number of graphs containing no K_4-pivot-minor? Is a graph without $\{K_4, W_5\}$-pivot-minor 3-colorable?

Problem

Proposed by Jinha Kim, Chonnam National University

- Let $\mathcal{F} = \{I_1, I_2, \ldots, I_n\}$ be a family of sets. We say R is a *rainbow set* of $\mathcal{F}$ if there is an injective map $\phi : R \to [n]$ such that $r \in I_{\phi(r)}$ for every $r \in R$.
- For a graph G, if $\mathcal{F}$ is a family of independent sets of G, then a vertex subset R is called a *rainbow independent set* if R is a rainbow set and an independent set.

Let $\mathcal{C}$ be a class of graphs. We define $f_{\mathcal{C}}(n)$ be the smallest number k that satisfies the following: For every graph $G \in \mathcal{C}$ and for every family $\mathcal{F} = \{I_1, I_2, \ldots, I_k\}$ where each I_i is an independent set of G with size n, there is a rainbow set R with size n.

Let $\mathcal{D}(k)$ be the class of all graphs with maximum degree at most k. In [1], it was conjectured that

$$f_{\mathcal{D}(k)}(n) = \left\lceil \frac{k+1}{2} \right\rceil (n-1) + 1.$$

This conjecture was partially proved when $k \leq 2$ or $n \leq 3$.

References

[1] R. Aharoni, J. Briggs, J. Kim, and M. Kim, Rainbow independent sets in certain classes of graphs, *J. Graph Theory*, 104(3):557–584, 2023.

Problem

Proposed by Minki Kim, GIST

In [1], the following variant of the Tverberg's theorem was proved: Let $d \geq 1$, $r \geq 2$, and let $n = (r-1)(d+1)$. Let $A \subset \mathbb{R}^d$ be a finite set of points of size larger than n. Then, there exists a partition $A = A_1 \cup \ldots \cup A_r$ and $B \subset A$ with $|B| = n$ such that for every $p \in A \setminus B$, the $\bigcap_{i\in[r]} \mathrm{conv}(A_i \cap B_p) \neq \emptyset$ where $B_p = B \cup \{p\}$.

It is a natural question if the above statement has a topological generalization. Here is the first case in this direction:

Conjecture. *Let $d \geq 1$, $n = d+1$ and $N > n$. For every continuous map from a simplex Δ on $N+1$ vertices to $\mathbb{R}^d$, there are two disjoint simplices F_1, F_2 and an $(n-1)$-dimensional simplex F of Δ such that for every $H \in \Delta$ with $|H| = d+2$, $f(F_1 \cap H) \cap f(F_2 \cap H) \neq \emptyset$.*

References

[1] Minki Kim and Alan Lew, Extensions of the colorful Helly theorem for d-collapsible and d-Leray complexes, arXiv:2305.12360.

Extremal numbers and Sidorenko's conjecture

Proposed by Joonkyung Lee, Yonsei University

Sidorenko's conjecture states that, for all bipartite graphs H, quasirandom graphs contain asymptotically the minimum number of copies of H taken over all graphs with the same order and edge density. While still open for graphs, the analogous statement is known to be false for hypergraphs. We show that there is some advantage in this, in that if Sidorenko's conjecture does not hold for a particular r-partite r-uniform hypergraph H, then it is possible to improve the standard lower bound, coming from the probabilistic deletion method, for its extremal number $\mathrm{ex}(n, H)$, the maximum number of edges in an n-vertex H-free r-uniform hypergraph. With this application in mind, we find a range of new counterexamples to the conjecture for hypergraphs, including all linear hypergraphs containing a loose triangle and all 3-partite 3-uniform tight cycles.

Properly colored spanning trees in edge-colored complete graphs without monochromatic triangles

Proposed by Ruonan Li, Northwestern Polytechnical University

Burr proved that each edge-colored complete graph without containing monochromatic triangle contains a properly colored Hamilton path. We propose the following question.

Question. *For each integer k, is there a constant $N(k)$ such that for every integer $n \geq N(k)$ and every n-vertex tree T with maximum degree at most k, each edge-colored K_n without containing monochormatic triangle contains a properly edge-colored T ?*

Problem

Proposed by Bingyu Luan, Shandong University & IBS ECOPRO

For a graph G, let $\chi(G)$ denote its chromatic number and $\sigma(G)$ denote the order of the largest clique subdivision in G. A famous conjecture of Hajós from 1961 states that $\sigma(G) \geq \chi(G)$ for every graph G and it was disproved by Catlin [2] in 1979.

Let $H(n)$ be the maximum of $\chi(G)/\sigma(G)$ over all n-vertex graphs G. Erdős and Fajtlowicz [3] showed by considering a random graph that $H(n) \geq cn^{1/2}/\log n$ for some absolute constant $c > 0$, and they conjectured that this bound is tight up to a constant factor in that there is some absolute constant C such that $\chi(G)/\sigma(G) \leq Cn^{1/2}/\log n$ for all n-vertex graphs G. In 2011, Fox, Lee and Sudakov [4] proved this conjecture. Using the relationship between chromatic number and independence number, they transform the problem into estimating the order of the largest clique subdivision which one can find in every graph on n vertices with independence number α. In addition, they gave the following conjecture.

Conjecture ([4]). *There is a constant $c > 0$ such that every graph G with chromatic number $\chi(G) = k$ satisfies $\sigma(G) \geq c\sqrt{k \log k}$.*

The bound in the above Conjecture would be best possible by considering a random graph of order $O(k \log k)$. The result of Bollobás and Thomason [1] and Komlós and Szemerédi [5] which says that every graph G of average degree d satisfies $\sigma(G) = \Omega(\sqrt{d})$. This is enough to imply the bound $\sigma(G) = \Omega(\sqrt{k})$ for G with $\chi(G) = k$, but not the extra logarithmic factor.

References

[1] B. Bollobás, A. Thomason, Proof of a conjecture of Mader, Erdős and Hajnal on topological complete subgraphs. *European Journal of Combinatorics*, 19, (1998), 883–887.

[2] P. Catlin, Hajós' graph-coloring conjecture: variations and counterexamples. *Journal of Combinatorial Theory, Series B*, 26, (1979), 268–274.

[3] P. Erdős, S. Fajtlowicz, On the conjecture of Hajós. *Combinatorica*, 1, (1981), 141–143.

[4] J. Fox, C. Lee, B. Sudakov, Chromatic number, clique subdivisions, and the conjectures of Hajós and Erdős-Fajtlowicz. *Combinatorica*, 33(2), (2013), 181–197.

[5] J. Komlós, E. Szemerédi, Topological cliques in graphs II. *Combinatorics, Probability and Computing*, 5, (1996), 79–90.

Problem

Proposed by Jie Ma, University of Science and Technology of China

For a graph H, let $m(H, k)$ denote the number of vertices with degree k in H. Alon and Wei made the following conjecture.

Conjecture. *Let G be an n-vertex d-regular graph. Then there exists a spanning subgraph H such that for any $0 \leq k \leq d$, $|m(H, k) - n/(d+1)| \leq 2$.*

Fox-Luo-Pham proved that if $d = o(n/(\log n)^{12})$, then there exists a spanning subgraph H such that $m(H, k) = (1 + o(1))n/(d + 1)$ for any $0 \leq k \leq d$. With Xie recently we prove this conjecture in the case $d = 3$; and in the general case, we prove "d^2" instead of "2". It is open for $d \geq 4$.

Problem

Proposed by Jaeseong Oh, Yonsei University

The *Kreweras number* for a partition λ is defined by

$$\mathrm{Krew}(\lambda) := \frac{1}{n+1}\binom{n+1}{n+1-\ell(\lambda), m_1(\lambda), \ldots, m_n(\lambda)},$$

where $n = |\lambda|$ and $m_j(\lambda)$ denotes the multiplicity of j among the parts of λ. Kreweras originally interpreted $\mathrm{Krew}(\lambda)$ in terms of the number of *noncrossing partitions* of $1, 2, \ldots, n$. Equivalently, Kreweras numbers count the number of Dyck paths whose lengths of consecutive horizontal steps are listed in λ. In the work of Kim, Lee, and myself we showed that

$$\sum_{\lambda \vdash k-1} (-1)^{k-1-\ell(\lambda)} \mathrm{Krew}((\lambda + (1^{\ell(\lambda)}))\binom{n}{\lambda + (1^{n+1-k})} = \frac{n!}{k}. \tag{1}$$

The proof uses intricate arguments using the inclusion-exclusion principle.

Question. *Is there a 'bijective' proof for* (1)*?*

It is worth note that there are similar combinatorial identity involving Kreweras numbers

$$\sum_{\lambda \vdash n} (-1)^{n-\ell(\lambda)} \mathrm{Krew}(\lambda + (1^{\ell(\lambda)}))\binom{n}{\lambda} = (n+1)^{n-1}.$$

Willows

Proposed by Sang-il Oum, IBS DIMAG & KAIST

A *directed tree* is an orientation of a tree. For a positive integer n, a graph G is an *n-willow* if there exists a directed tree T with $V(G) \subseteq V(T)$ such that for every distinct pair u, v of vertices of G, the vertices u and v are adjacent if and only if T has a directed path from u to v or from v to u whose length is not a multiple of n. In this case, we say G is an n-willow defined by T. Chudnovsky, Cook, Davies, and myself showed that several graph classes are not willows.

Question:

1. Characterize the class of all willows by a list of forbidden induced subgraphs.
2. Prove that if H is a willow, then for every χ-bounded class $\mathcal{F}$ of graphs, the class of H-free graphs in $\mathcal{F}$ is polynomially χ-bounded.

For the second problem, Chudnovsky, Cook, Davies, and myself showed the converse.

Note that a class $\mathcal{F}$ of graphs is χ-bounded if there is a function f such that $\chi(H) \leq f(\omega(H))$ for all induced subgraphs H of a graph in $\mathcal{F}$. If f can be chosen to be a polynomial, we say that $\mathcal{F}$ is polynomially χ-bounded.

What's known for the problem 1: All pentagram spiders, all tall striders, all short striders, the complement $\overline{P_9}$ of P_9, and the complement $\overline{C_n}$ of C_n for all $n \geq 7$ are not willows. We also know that C_5, $\overline{C_6}$, and $\overline{P_8}$ are willows.

A 10-vertex graph G is a *pentagram spider* if it has a perfect matching M such that $G \setminus M$ has a component isomorphic to K_5. A 12-vertex graph is a *tall strider* if it has a clique $C = \{x_1, x_2, x_3\}$ of size 3 such that $N(x_1) \setminus C$, $N(x_2) \setminus C$, and $N(x_3) \setminus C$ are disjoint cliques of size 3. A 10-vertex graph is a *short strider* if it has a clique $C = \{x_1, x_2, x_3, x_4\}$ of size 4 such that $N(x_1) \setminus C$, $N(x_2) \setminus C$, and $N(x_3) \setminus C$ are disjoint cliques of size 2.

References

[1] Maria Chudnovsky, Linda Cook, James Davies, and Sang-il Oum, Reuniting χ-boundedness with polynomial χ-boundedness, 2023.

Problem

Proposed by Yuejian Peng, Hunan University

$\alpha \in [0,1)$ is a *jump* for r if $\exists\ c > 0$ such that $(\alpha, \alpha + c) \cap \Pi_r = \emptyset$, where Π_r is the set of Turán densities of all families of r-uniform hypergraphs. A classical result of Erdős-Stone-Simonovits implies that every $\alpha \in [0,1)$ is a jump for $r = 2$. It was shown by Erdős that every number in $[0, \frac{r!}{r^r})$ is a jump for $r \geq 3$.

Question (Erdős, 1964). *Is* $\dfrac{r!}{r^r}$ *a jump for* $r \geq 3$*? What is the smallest non-jump for* $r \geq 3$*?*

Question (Frankl-Peng-Rödl-Talbot, 2007). *Is there* $a_r \in (0,1)$ *such that no value in the interval* $(a_r, 1)$ *is a jump for* $r \geq 3$*?*

Pikhurko showed that the set $\Pi^r \subseteq [0,1]$ is closed for $r \geq 3$. So this question is equivalent to whether Π_r contains an interval for $r \geq 3$.

Problem

Proposed by Bruce Reed, Institute of mathematics, Academia Sinica

By a k by k grid we mean a checkerboard whose vertices are labelled from $\{1, ..., k\}^2$ and where two are adjacent if they agree in one coordinate and differ by one in the other.

A fundamental theorem of Robertson and Seymour tells us that for every k there is an $f(k)$ such that if G does not contain a k by k grid as a minor then it has tree width at most $f(k)$. For general k the value of $f(k)$ is quite large, although Chekuri and Chuzoy improved the exponential bound on $f(k)$ of Robertson and Seymour to one which is bounded by a polynomial in k.

Birmele, Bondy, and Reed proved that $f(3) = 8$. They also showed $f(4) \leq 7262$. See

`https://link.springer.com/content/pdf/10.1007/978-3-7643-7400-6_4.pdf`

the best lower bound on $f(4)$ is near 16. Can these bounds on $f(4)$ be improved?

The following recent paper is possibly relevant:

`https://onlinelibrary.wiley.com/doi/epdf/10.1002/jgt.22911`

Problem

Proposed by Chong Shangguan, Shandong University

We will discuss two combinatorial problems in the study of expander (Tanner) codes. A bipartite graph G with bipartition $L \cup R$ is called a (c, d, α, δ)-bipartite expander if it is (c, d)-regular and for every subset $S \subseteq L$ with $|S| \leq \alpha n$, $|N(S)| \geq \delta c|S|$. The expander code $C(G) \subseteq \mathbb{F}_2^n$ is defined as

$$C(G) = \{(x_1, \ldots, x_n) \in \mathbb{F}_2^n : \sum_{v \in N(u)} x_v = 0 \text{ for every } u \in R\}.$$

The landmark work of Sipser and Spielman showed that every bipartite expander G with expansion ratio $\delta > 3/4$ defines an asymptotically good (expander) code with linear-time decoding algorithm. Viderman showed that $\delta > \frac{2}{3} - \frac{1}{6c}$ is sufficient and $\delta > \frac{1}{2}$ is necessary.

Question. *Is $\delta > \frac{1}{2}$ also sufficient?*

To construct graph codes with weaker requirement on the expansion ratio, let us consider the following generalization of expander codes. Let $C_0 \subseteq \mathbb{F}_2^n$ be a linear subspace. The minimum distance d_0 of C_0 is the minimum weight among all nonzero vectors in C_0. The Tanner code $T(G, C_0)$ is defined as

$$T(G, C_0) = \{(x_1, \ldots, x_n) \in \mathbb{F}_2^n : x_{N(u)} \in C_0 \text{ for every } u \in R\},$$

where $x_{N(u)}$ denotes the restriction of x to the coordinates in $N(u)$.

Dowling and Gao showed that every bipartite expander G with expansion ratio $\delta = \Omega(\sqrt{c/d_0})$ defines an asymptotically good (Tanner) code with linear-time decoding algorithm. Recently, Cheng, Shangguan, and Shen showed that $\delta = \Omega(1/d_0)$ is sufficient and $\delta > 1/d_0$ is necessary.

Question. *Is $\delta > \frac{1}{d_0}$ also sufficient?*

Infinite series with summands containing binomial coefficients and harmonic numbers

Proposed by Zhi-Wei Sun, Nanjing University

For $m = 1, 2, 3, \ldots$, the harmonic numbers of order m are given by

$$H_n^{(m)} := \sum_{0<k\le n} \frac{1}{k^m} \quad (n = 0, 1, 2, \ldots).$$

In this talk, we first give a brief survey of infinite series identities with summands containing both binomial coefficients and harmonic numbers, and then focus on various open conjectures on this topic posed by the speaker.

For example, in 1993 D. Zeilberger [Contemporary Math. 143 (1993), 579-607] used the WZ method to establish the identity

$$\sum_{k=1}^{\infty} \frac{21k-8}{k^3\binom{2k}{k}^3} = \frac{\pi^2}{6},$$

and we conjecture the following identities:

$$\sum_{k=1}^{\infty} \frac{21k-8}{k^3\binom{2k}{k}^3}\left(H_{2k-1}^{(3)} + \frac{43}{8}H_{k-1}^{(3)}\right) = \frac{711}{28}\zeta(5) - \frac{29}{14}\pi^2\zeta(3),$$

$$\sum_{k=1}^{\infty} \frac{(21k-8)H_{k-1}^{(3)} + 1/k^2}{k^3\binom{2k}{k}^3} = \frac{62}{7}\zeta(5) - \frac{16}{21}\pi^2\zeta(3),$$

$$\sum_{k=1}^{\infty} \frac{9(21k-8)H_{k-1}^{(4)} + 25/k^3}{k^3\binom{2k}{k}^3} = \frac{13\pi^6}{3780}.$$

In 2013 the speaker [Electron. J. Combin. 20 (2013), no. 1, # P9] conjectured that

$$\sum_{k=1}^{\infty} \frac{(28k^2-18k+3)(-64)^k}{k^5\binom{2k}{k}^4\binom{3k}{k}} = -14\zeta(3),$$

which was confirmed by K.C. Au [arXiv:2212.02986] in 2022, in contrast we conjecture that

$$\sum_{k=1}^{\infty} \frac{(-64)^k}{k^5\binom{2k}{k}^4\binom{3k}{k}}\left((28k^2-18k+3)(4H_{2k-1}^{(3)} + 3H_{k-1}^{(3)}) + \frac{4}{k}\right) = -49\zeta(3)^2.$$

Problem

Proposed by Leo Versteegen, University of Cambridge

The set of graphs with vertex set $[n]$ can be made into a vector space over $\mathbb{F}_2$ by defining the sum of two graphs $G_1 = ([n], E_1)$ and $G_2 = ([n], E_2)$ as the graph on $[n]$ whose edge set is the symmetric difference of E_1 and E_2. We say that a graph $G \in \mathbb{F}_2^{K_n}$ is a *copy* of a graph H if G contains a set S such that the induced subgraph $G[S]$ is isomorphic to H and all vertices in $[n] \setminus S$ are isolated. A subset $\mathcal{C} \subset \mathbb{F}_2^{K_n}$ is called a graph code, and we say that it is a *linear graph code* if $\mathcal{C}$ is a subspace. Recently, Alon suggested to investigate the quantities

$$d_H(n) = 2^{-\binom{n}{2}} \max\{|\mathcal{C}| : \text{No sum of two graphs in } \mathcal{C} \text{ is a copy of } H\}$$

and

$$d_H^{\text{lin}}(n) = 2^{-\binom{n}{2}} \max\{|\mathcal{C}| : \mathcal{C} \text{ is a linear graph code and contains no copy of } H\}.$$

Because linear graph codes are closed under addition, $d_H^{\text{lin}}(n) \leq d_H(n)$ for all H and n, but to my best knowledge, the following question is open.

Question: Does there exist a graph H such that $d_H(n) > d_H^{\text{lin}}(n)$ for infinitely many $n \in \mathbb{N}$?

Problem

Proposed by Guanghui Wang, Shandong University

Given two graphs G and H, a TH-packing in G is a collection of vertex-disjoint subdivisions of H in G. It is perfect if all of the vertices of G are covered. Verstraëte proposed the following conjecture.

Conjecture. *For every graph H and every positive ε, there exists an integer $r_0 = r_0(H, \varepsilon)$ such that, for all $r \geq r_0$, every r-regular graph G contains a TH-packing which covers all but at most $\varepsilon|G|$ vertices of G.*

The conjecture was true for trees by Kelmans, Mubayi and Sudakov [2], and Alon [1] proved it for cycles. Kühn and Osthus [3] also provided further support for this conjecture.

Theorem ([3])**.** *Given a graph H without isolated vertices which is not a union of cycles and a constant $0 < c \leq 1$, there exist $\gamma = \gamma(H, c) > 0$ and $C = C(H, c)$ such that every $(cn \pm \gamma n)$-regular graph G has a TH-packing which covers all but at most C vertices of G.*

Complete bipartite graphs $K_{k,\ell}$ with $k - \ell = \gamma n$ shows that the above Theorem is not true if H is a union of cycles.

Problem. *Prove that the above Conjecture is true for H, where H is a union of cycles.*

In particular, if $H = K_4$ or $H = K_5$, then [3] gives a perfect packing in every almost regular graph.

Problem. *Given $r \geq 6$ and $0 < c < 1$, does every cn-regular graph of sufficiently large order n have a perfect TK_r-packing?*

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[2] A. Kelmans, D. Mubayi, B. Sudakov, Asymptotically optimal tree-packings in regular graphs, *Electronic J. Combinatorics*, 8:R38, 2001.

[3] D. Kühn, D. Osthus, Packings in dense regular graphs, *Combinatorics, Probability and Computing*, 14:325-337, 2005

Problem

Proposed by Baogang Xu, Nanjing Normal University

A hole of a graphs is an induced cycle of length at least 4. An odd hole is a hole of length odd. Let l be a positive integer, and let $\mathcal{G}_l$ be the set of graphs of girth at least $2l+1$ and without odd hole of length at least $2l+3$. We will introduce some results and open questions on $\cup_{\geq 2}\mathcal{G}_l$ and related graphs.

Combinatorial proof of Graham-Pollak theorem

Proposed by Zixiang Xu, IBS ECOPRO

The famous Graham-Pollak theorem states that the edges of a $(d + 1)$-vertex complete graph cannot be covered by fewer than d bi-cliques. In other words, the maximum number of vertices in a complete graph whose edges can be covered at least once by d bi-cliques is $d+1$.

The theorem has since become well known and repeatedly studied and generalized in graph theory, in part because of its elegant proof using techniques from algebraic graph theory. More strongly, Aigner and Ziegler write that all proofs are somehow based on linear algebra: "no combinatorial proof for this result is known." I will introduce some proofs and generalizations of the above theorem.

It is time to provide a purely combinatorial proof!!

Whether any matroid Kazhdan-Lusztig polynomial has only real zeros?

Proposed by Libo Yang, Nankai University

Elias, Proudfoot, and Wakefield [1] introduced the notion of the Kazhdan-Lusztig polynomial of a matroid. Given a loopless matroid M, let $L(M)$ denote the lattice of flats of M, let $\chi_M(t)$ denote its characteristic polynomial, and let $\operatorname{rk} M$ denote the rank of M. They proved that there is a unique way to associate to each M a polynomial $P_M(t) \in \mathbb{Z}[t]$ satisfying the following properties:

- If $\operatorname{rk} M = 0$, then $P_M(t) = 1$.
- If $\operatorname{rk} M > 0$, then $\deg P_M(t) < \frac{1}{2} \operatorname{rk} M$.
- For every M, $t^{\operatorname{rk} M} P_M(t^{-1}) = \sum_{F \in L(M)} \chi_{M_F}(t) P_{M^F}(t)$,

where the symbol M^F represents the contraction of M at F, and M_F represents the localization of M at F.

Gedeon, Proudfoot, and Young [2] proposed the following conjecture.

Conjecture. *For any matroid M, the polynomial $P_M(t)$ has only real zeros.*

In this talk I will discuss some progress on this conjecture.

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Ennola duality for orthogonal groups

Proposed by Semin Yoo, Korea Institute for Advanced Study

Ennola duality is a phenomenon that the character table of $U(n, q^2)$ is obtained by the character table of $GL(n, q)$, roughly speaking, by replacing q by $-q$. Ennola (1963) proved it for $n \leq 3$. After some partial results were done, Kawanaka (1985) proved it for all n. Since then, people have been studying various Ennola types of duality. I guess that the character table of $O(n, q)$ when $q \equiv 1 \pmod 4$ can be obtained by the ones when $q \equiv 3 \pmod 4$, which shows that there is another Ennola type of duality from orthogonal groups. This guess is based on the combinatorial phenomenon in my recent paper about Euclidean binomial coefficients.

Problem

Proposed by Long-Tu Yuan, East China Normal University
and Xiao-Dong Zhang, Shanghai Jiao Tong University

In 1959, Erdős and Gallai proved the following theorem.

Theorem. *Let G be a simple graph with avedeg$(G) > k-2$. Then G contains a path of order k.*

Further Erdős and Sós proposed the following well known conjecture

Conjecture. *Let G be a simple graph with avedeg$(G) > k-2$. Then G contains all trees of order k.*

For bipartite extremal graphs, we proposed the following conjecture.

Conjecture. *Let $n \geq m, k \geq l, n \geq k$ and $m \geq l$.*

(i) *Let G be an (n,m)-bipartite graph with $e(G) > (l-1)(n+m-2l+2)$. If $k < 2l-2$ and $m \geq 2l$, then G contains all (k,l)-bipartite tree.*

(ii) *Let G be an (n,m)-bipartite graph with $e(G) > (k-1)(m-l+1)+(l-1)(n-m+l-1)$. If $k \geq 2l-1$ and $m-l+1 \geq k-1, n-m+l-1 \geq k$, then G contains all (k,l)-bipartite tree.*

(iii) *Let G be an (n,m)-bipartite graph with $e(G) > (k-1)m$. If $k \geq 2l-1$ and $m-l+1 \geq k-1, n-m+l-1 \leq k-1$, then G contains all (k,l)-bipartite tree.*

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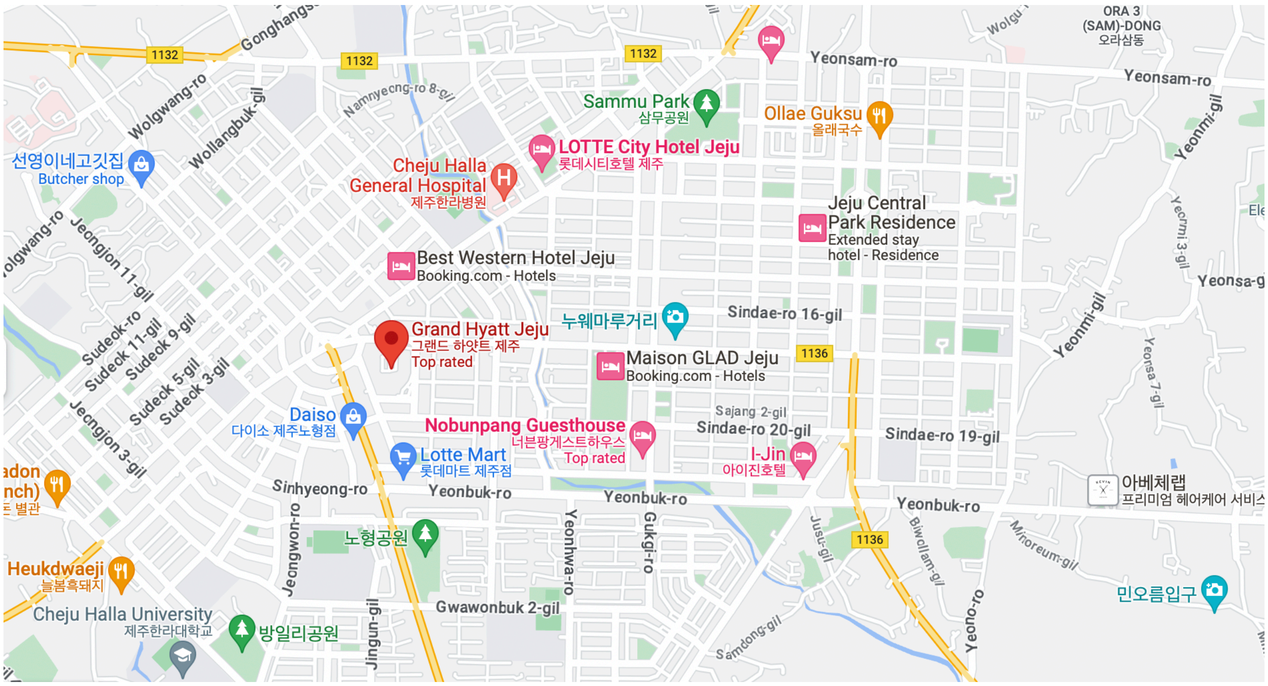
How to get from the airport to Grand Hyatt Jeju

Grand Hyatt Jeju is located 4km from Jeju International airport: 10 minute by car; 6km from Jeju International ferry terminal: 15 minute by car. Duty free shops, local markets, shopping streets, including restaurants and bars are located within close proximity.



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Bus Stop near hotel: Alight at Wonnohyeong (approx. 2 minutes walk):
Bus Service Numbers:
Bus No 316 – Alight on the 7th stop from airport
Bus No 465 – Alight on the 7th stop from airport
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