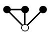
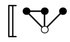
Graph limits and flag algebras
Exercises – set 2

Problem 1. Calculate $d($$,$ $)$,  $\cdot$  and $[\![$$]\!]$.

Problem 2. Using the inequality $[\![($ $-$ $)^2]\!] \geq 0$ prove that  $+$  $\geq 1/4$.

Problem 3. Consider a convergent sequence of graphs, where vertices of each n-vertex graph are of degree $n/3 + o(n)$ or $2n/3 + o(n)$. Prove that the limiting graphon satisfies  $+$  $= 1/3$.

Problem 4. Using the inequality $[\![$$^2]\!] \geq [\![$$]\!]^2$ prove that  $\geq$ $(2$ $- 1)$.

Problem 5. Using the previous problem, show that any graph on n vertices and e edges contains at least $e\left(4e - n^2\right)/3n$ triangles.

Problem 6. Prove the inequality  ≤ 3 $+ 3/8$ and argue that any extremal graph satisfies  $= 1/4$ for almost all possible placements of the root.

Problem 7. Prove that $^2 \leq$ ^{3} and deduce the extremal examples.

Problem 8. Prove that for every integer $k \geq 2$ it holds $K_{k+1} \geq (k$ $- (k-1))K_k$. Using it deduce Turán's theorem that any n-vertex graph without K_{k+1} contains at most $\left(1 - \frac{1}{k}\right)\frac{n^2}{2}$ edges.