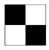
Graph limits and flag algebras
Exercises – set 1

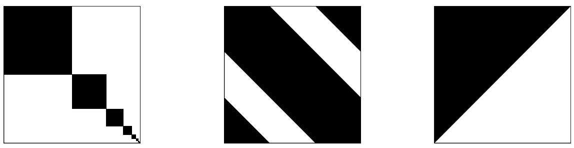
Problem 1. Prove that the sequence $(K_{n,2n})_{n\in\mathbb{N}}$ is convergent.

Problem 2. Let G_n be a graph containing a clique on p vertices and $n-p$ isolated vertices, where p is the smallest prime divisor of n. Decide if the sequence $(G_n)_{n\in\mathbb{N}}$ is convergent.

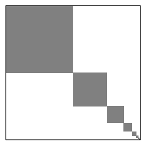
Problem 3. Prove that the following is true for any sequence of graphs $(G_n)_{n\in\mathbb{N}}$ of growing orders: for every graph H the sequence of densities $d(H, G_n)$ is convergent if and only if for every graph H the sequence of homomorphism densities $t(H, G_n)$ is convergent

Problem 4. Determine d ( , ).

Problem 5. What sequences of graphs can converge to the following graphons?



Problem 6. Consider a real number $p \in [0,1]$ and a graphon W such that $W(x,y) = p$ if $x, y \in [1-2^{-n+1}, 1-2^{-n}]$ for some integer $n \geq 1$, and $W(x,y) = 0$ otherwise. For any integer $k \geq 2$ determine the density of K_k in W.



Problem 7. Express $d($  $, W')$ in terms of $d($  $, W)$, $d($  $, W)$, $d($  $, W)$ and $d($  $, W)$, where W' is the depicted graphon containing two equal-sized parts, one with the graphon W and one with the complete graphon.